

Mid term Exam :- 30 marks

Internal :- 20 marks (will be increased to 30)

End term :- 50 marks (will be reduced to 40)

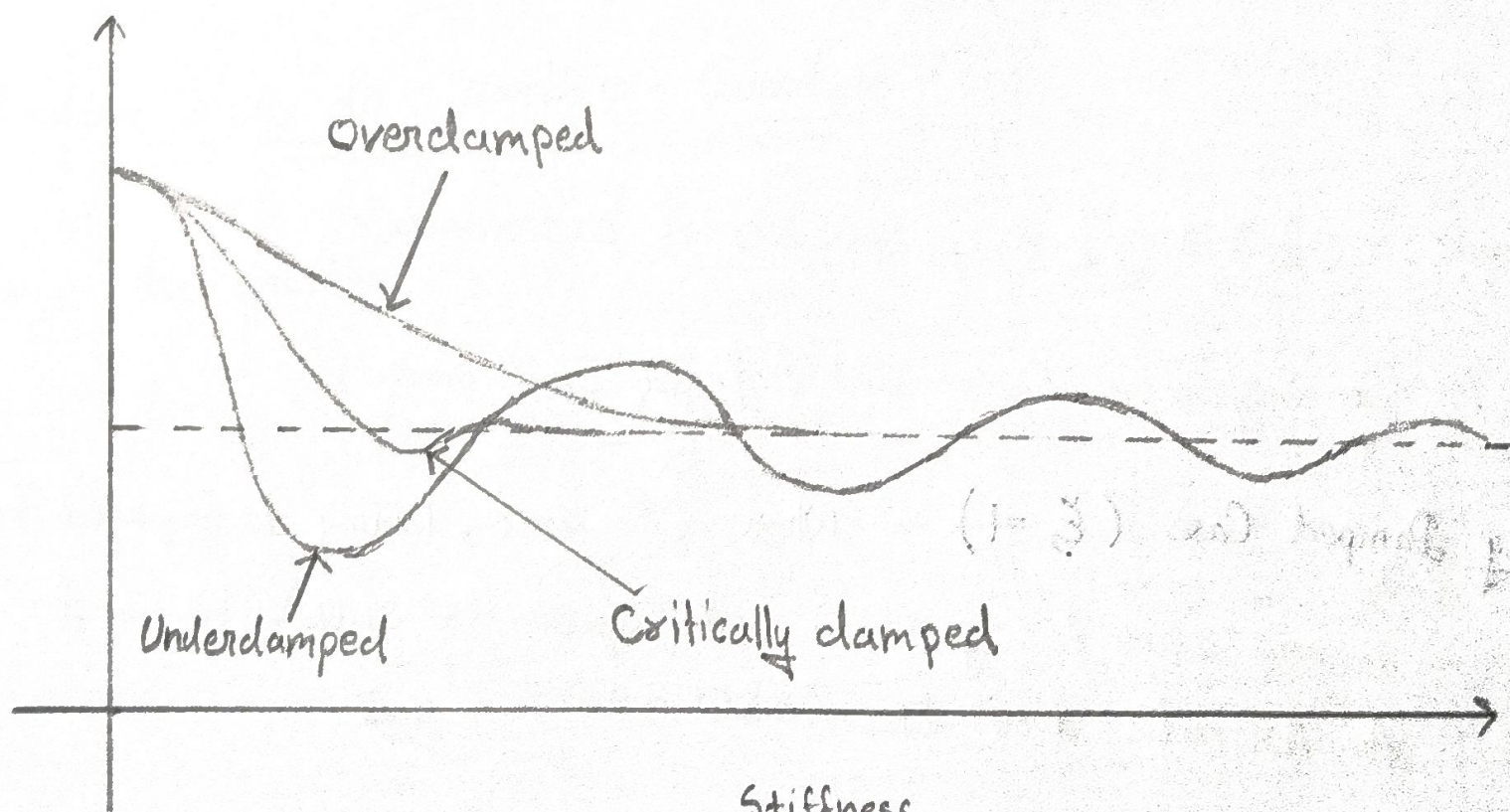
(minimum two coding Assignment will be given)

Flight Dynamics and Control - AE614

Transient Behaviour:- The temporary response often short lived of a system as it shifts from one steady state to another.

The spring mass damper system is an example of transient behaviour.

Spring Mass Damper System:-



$$\Rightarrow m\ddot{x} + c\dot{x} + Kx = F(x,t)$$

$\uparrow$ mass $\uparrow$ damping Coefficient $\downarrow$ Stiffness $\uparrow$ Displacement

Without perturbation or any other force,

$$m\ddot{x} + c\dot{x} + kx = 0$$

Standard form of the characteristic equation :-

$$s^2 + 2 \underset{\substack{\uparrow \\ \text{Damping} \\ \text{ratio}}}{\xi} \overset{\substack{\downarrow \\ \text{natural frequency}}}{\omega_n} s + \omega_n^2 = 0$$

Based on the value of the damping ratio (ξ), the System exhibits different behaviours,

① Underdamped ($0 < \xi < 1$) — roots are complex conjugates with negative real part.

(i) Behaviour :- Oscillatory, Amplitude decays exponentially.

(iii) Example :- Car Suspension System after hitting a small bump.

② Critically Damped Case ($\xi = 1$) — When $C^2 = 4mk$, leading to repeated real roots.

(i) Behaviour — fastest return to equilibrium, the System is stable.

(ii) Example :- An Automatic door Closer.

③ Overdamped Case ($\xi > 1$) — When the damping is very high ($C^2 > 4mk$), the roots are real, distinct & negative.

(i) Behaviour :- no Oscillation, return to equilibrium Slow & Sluggish

(ii) Example :- The door with with very heavy stiff hydraulic Closer.

Unstable Cases :

✓ Negative Stiffness ($k < 0$) : This results in real roots with opposite signs.
Causing one exponential term to grow with time.
making the system physically unstable.

✓ Negative Damping ($c < 0$) : In this case, energy is added to the system rather than dissipated, causing displacements to grow exponentially.

Acceleration due to earth's Curvature (Geometric effect) :

✓ Change in gravity

$$\Delta g = -\frac{2g_0 h}{R}$$

for $h = 10 \text{ km}$,

$$\Delta g = -0.03 \text{ m/s}^2$$

→ Much larger than Centrifugal change

→ Reduces gravity magnitude

→ Alters aircraft equilibrium and required lift.

So, Gravity Variation due to earth's Curvature,

at 15 km,

$$\Delta g_{15} = 9.764 \text{ m/s}^2$$

at 30 km,

$$\Delta g_{30} = 9.717 \text{ m/s}^2$$

{ The change
is dominant }

Change in Centrifugal acceleration due to earth's rotation,

at 15 km,

$$\Delta a_{ct, 15} = 8.0 \times 10^{-5} \text{ m/s}^2$$

at 30 km,

$$\Delta a_{ct, 30} = 1.6 \times 10^{-5} \text{ m/s}^2$$

{ The change
is negligible }

Reference Frame :-

① Inertial Reference frame :

$X_E, Y_E, Z_E \rightarrow \text{North, East, Down (N-E-D)}$

Its origin is at point of interest on the surface of the earth

$\rightarrow$ We have assumed that earth is flat.

Purpose :

$\rightarrow$ Describing position, velocity, and trajectory

$\rightarrow$ Applying Newton's laws

$\rightarrow$ Defining gravity

Used for :

$\rightarrow$ Range, Endurance, Trajectory

$\rightarrow$ Latitude, longitude, altitude

$\rightarrow$ long range navigation

Problem:

- Aircraft rotates and translates
- forces are not aligned with this frame.

② Local Horizontal Coordinate System: Also called Earth horizontal
Carried local level frame, navigation frame.

X_n, Y_n, Z_n → parallel to X_E, Y_E, Z_E (origin at C.G. of
the aircraft non-rotating
& accelerating)

Purpose :-

- Describing flight path relative to Earth.
- Defining flight angle γ and heading ψ
- Navigation, guidance and autopilot design
- Separating kinematics from aircraft altitude.

③ Body fixed Coordinate System :- attached to aircraft.

X_b, Y_b, Z_b → origin at C.G. Rotating & accelerating

→ This is the one which depicts the aircraft completely.

✓ Purpose :-

- Writing equations of motion
- Expressing forces and moments
- Control Analysis :

✓ Why required :-

- Forces from engines, control surfaces, inertia moments are fixed to the aircraft.
- Sensors (IMU, gyros) measure in body axes.
- Mass and inertia tensor are constant in body frame.
- It simplifies rotational dynamics.
- Essential for Stability & Control.

✓ Problem :-

- Aerodynamic forces are not aligned with axes & wind (Velocity) Coordinate System: body axes.

✱

④ Wind (Velocity) Coordinate :-

$X_w, Y_w, Z_w \rightarrow$ Origin at C.G. X_w along Relative V_∞

Purpose :-

$\rightarrow$ Describing lift, drag, and side force.

$\rightarrow$ Aerodynamic Coefficient modeling.

Why required :-

$\rightarrow$ Aerodynamic depends on relative airflow, not Earth or body orientation.

$\rightarrow$ lift is always $\perp$ to Velocity. Drag is always opposite to Velocity.

Problem :- Wind Axes rotate continuously with motion.

⑤ Stability Coordinate System :- (for Small disturbance Analysis)

$\rightarrow$ removes large Steady term

$\rightarrow$ Allows linear equations.

essential for :-

$\rightarrow$ longitudinal and lateral-directional stability.

$\rightarrow$ Eigenvalue Analysis.

✓ Why required :- motion is considered as small perturbations about trim.

→ Reference Velocity is fixed.

✓ Navigation / NED Coordinate System :-

✓ Purpose :- Avionics and guidance
GPS, INS integration.

✓ Why Required :-

→ Navigation algorithms operate in Earth referenced directions

⇒ Gravity is simply :-

$$\underline{g} = [0 \quad 0 \quad g]^T$$

✓ Used in :-

→ Autopilots

→ Flight management Systems

→ Vectors does not depend on the Coordinate System but the representation of the Vector will depend on the Coordinate System.

Euler's Theorem of Rotation

Statement:- Any change in Orientation of a rigid body in 3D Space can be represented as a Single rotation through some angle θ about a fixed axis.

This axis is called

a euler's axis.

→ This is true regardless of how Complicated the motion is

→ The axis passes through a fixed point (Usually the Center of mass)

Mathematical formulation:-

Let,

R be a rotation matrix (Orthogonal, $\det = 1$)

Then Euler's theorem states:-

$$R = I \cos \theta + (1 - \cos \theta) \hat{e} \hat{e}^T + \sin \theta [\hat{e}]_x$$

Where,

I = identity matrix

θ = rotation angle

$[\hat{e}]_x$ = Skew-Symmetric matrix

→ Euler angle (Yaw-Pitch-roll) :- ψ, θ, ϕ are one way of constructing the rotation axes, but : They are not the Euler axis. They are a sequence of rotation, not a single one

→ Euler theorem Says :-

$$R_z(\psi) R_y(\theta) R_x(\phi) \leftarrow \text{Single rotation about Euler axis.}$$

Important result :-

for a rotation matrix R :

Rotation angle :-

$$\theta = \cos^{-1} \left(\frac{\text{tr}(R) - 1}{2} \right)$$

Rotation axis :-

$$\hat{e} = \frac{1}{2 \sin \theta} \begin{bmatrix} R_{32} - R_{23} \\ R_{23} - R_{31} \\ R_{21} - R_{12} \end{bmatrix}$$

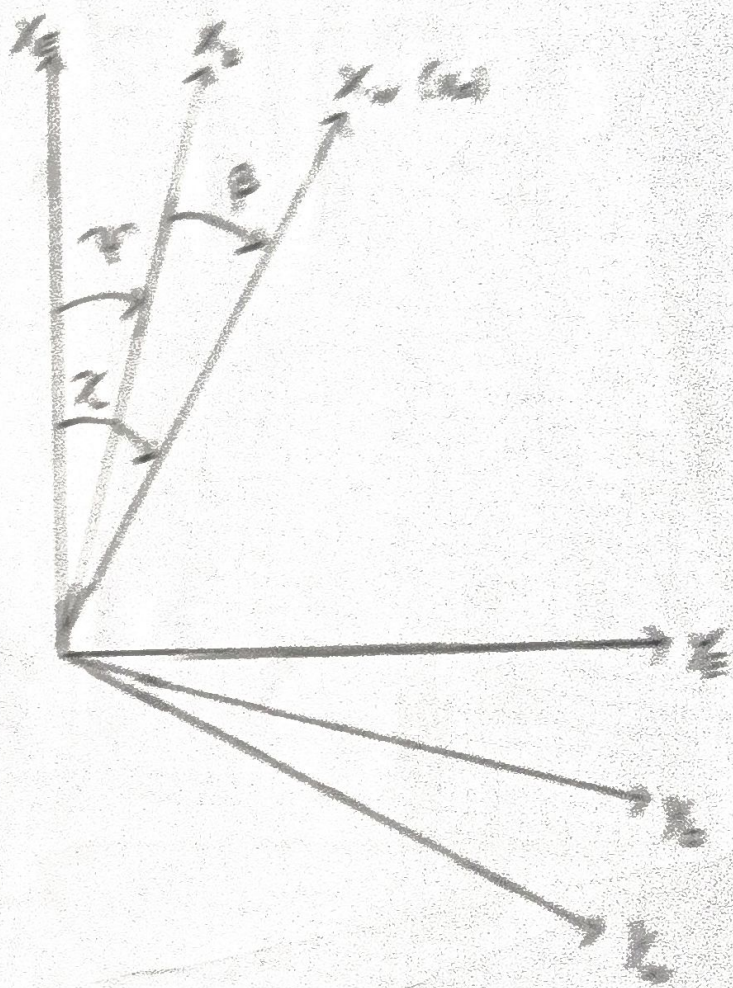
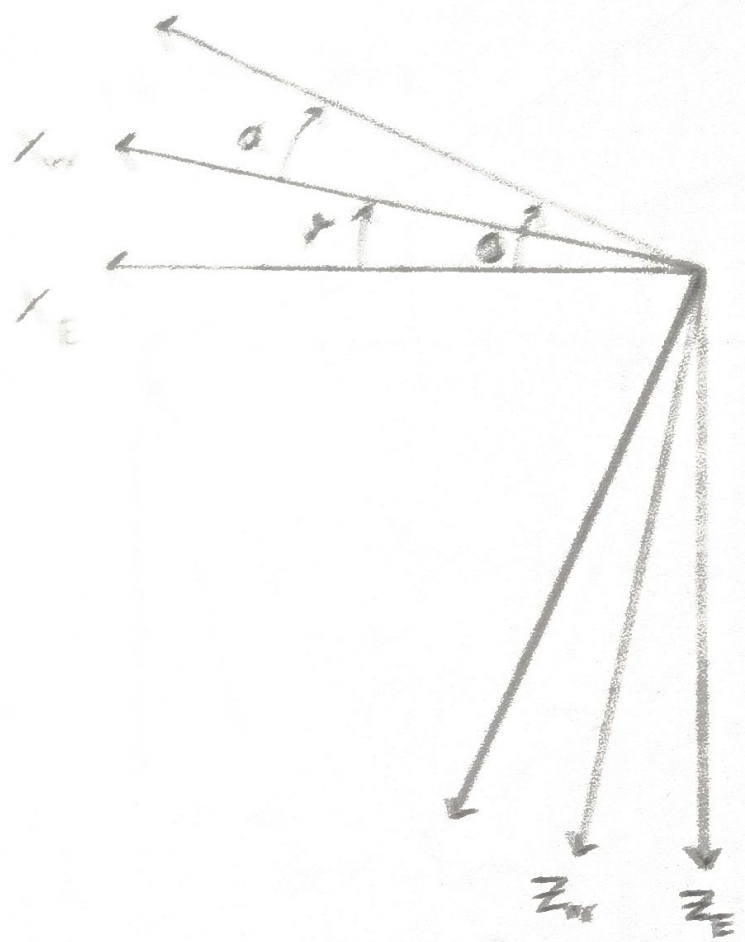
This comes directly from Euler theorem.

→ In body Axis System :-

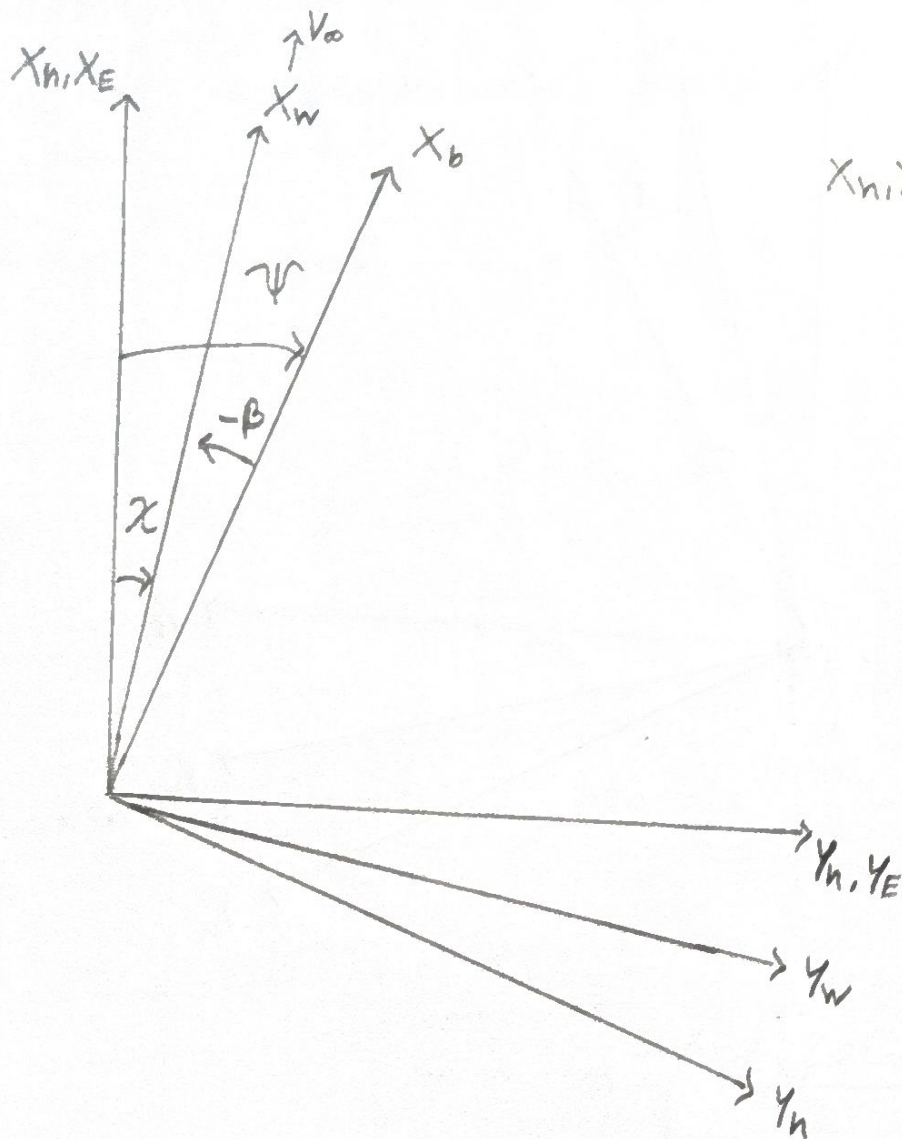
$X_b Z_b$ → longitudinal plane

$X_b Y_b$ → Directional plane

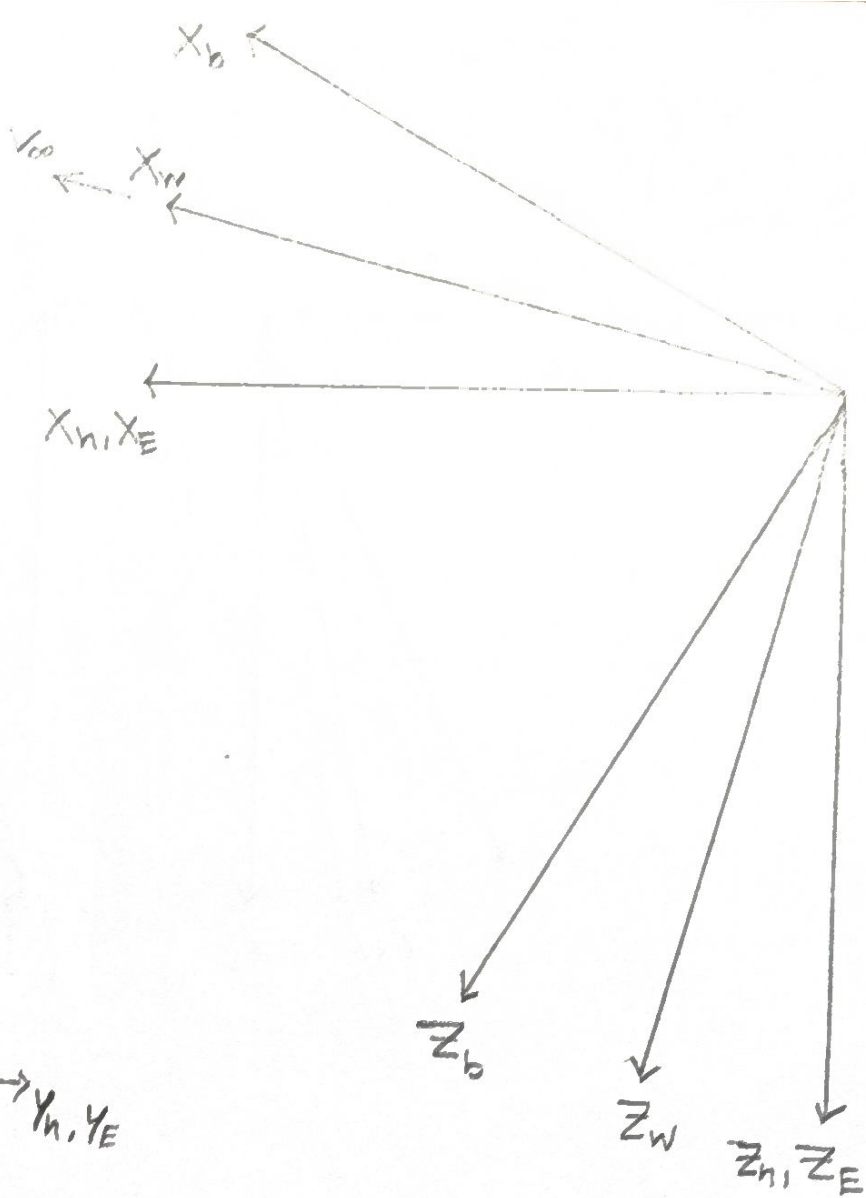
$Y_b Z_b$ → lateral plane



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$$\psi = \alpha + \beta$$



$$\begin{aligned} \theta &= \psi + \alpha \\ \dot{\theta} &= \dot{\psi} + \dot{\alpha} \end{aligned}$$

- Right wing going back is positive ψ .
- Right wing going down is positive roll.
- Pitch up is positive
- ⇒ $X_E, Y_E, Z_E \rightarrow X_b, Y_b, Z_b$

Inertial frame to $\xrightarrow{\psi \hat{z}_3} \xrightarrow{\theta \hat{y}_2} \xrightarrow{\phi \hat{x}_1} \text{Non-Inertial}$

first rotation about z axis
second rotation about y axis
third rotation about x axis

⇒ Transformation :-

⇒ $X_E, Y_E, Z_E \longrightarrow X_w, Y_w, Z_w$

⇒ $x \hat{z}_E - y \hat{y}' - z \hat{x}_w$

⇒ $X_w, Y_w, Z_w \longrightarrow X_b, Y_b, Z_b$

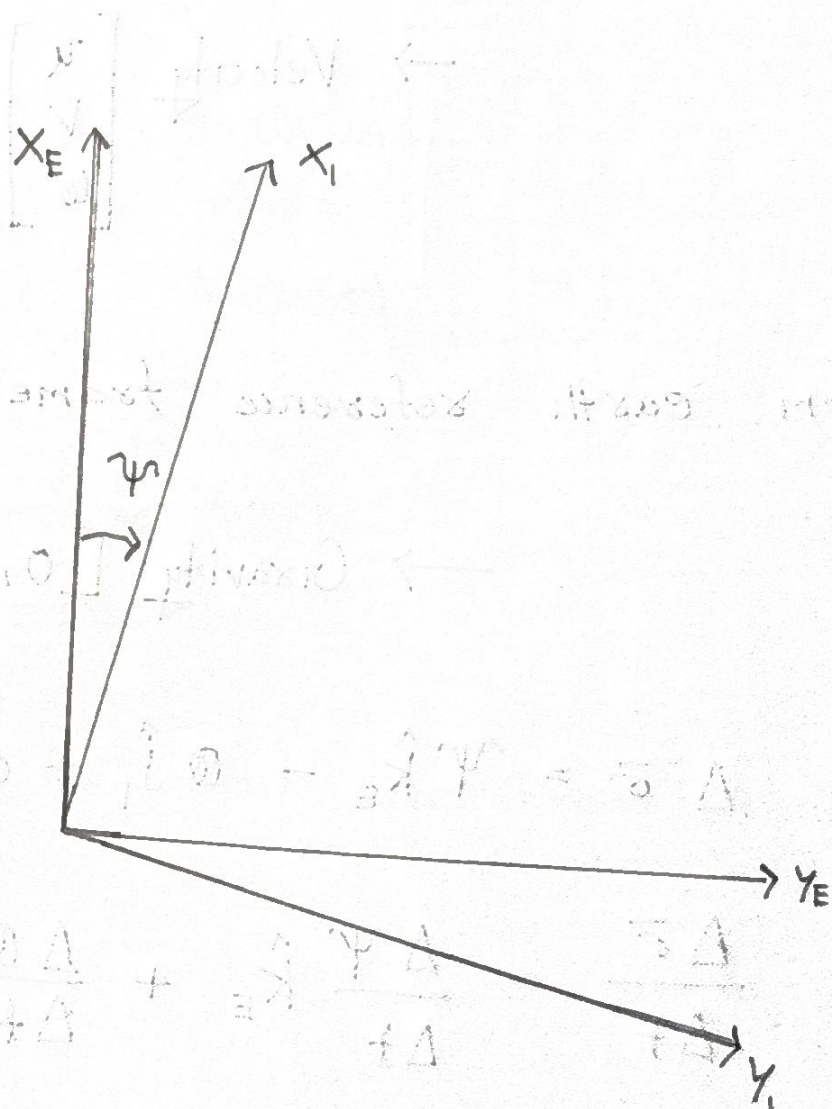
⇒ $- \beta \hat{z}_w - \alpha \hat{y}_s$

⇒
$$\begin{bmatrix} \hat{l}_E \\ \hat{j}_E \\ \hat{k}_E \end{bmatrix} = \underbrace{\begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_\psi^T} \begin{bmatrix} \hat{l} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

⇒
$$\begin{bmatrix} \hat{l}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}}_{R_\theta^T} \begin{bmatrix} \hat{l}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{bmatrix}$$

⇒
$$\begin{bmatrix} \hat{l}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}}_{R_\phi^T} \begin{bmatrix} \hat{l}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix}$$

⇒
$$\begin{bmatrix} \hat{l}_E \\ \hat{j}_E \\ \hat{k}_E \end{bmatrix} = R_\psi^T \cdot R_\theta^T \cdot R_\phi^T \cdot \begin{bmatrix} \hat{l}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix}$$



∴
$$\begin{bmatrix} \hat{l}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix} = R_\phi R_\theta R_\psi \begin{bmatrix} \hat{l}_E \\ \hat{j}_E \\ \hat{k}_E \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} \hat{i}_E \\ \hat{j}_E \\ \hat{k}_E \end{bmatrix} = R_\psi^T R_\theta^T R_\phi^T \begin{bmatrix} \hat{i}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix}$$

$\Rightarrow$ from body Reference frame to earth reference frame

$$\rightarrow \text{Velocity} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$\Rightarrow$ from earth reference frame to body reference frame

$$\rightarrow \text{Gravity} [0, 0, g]^T$$

$$\Delta \bar{\sigma} = \psi \hat{k}_E - \theta \hat{j}_1 - \phi \hat{i}_b$$

$$\frac{\Delta \bar{\sigma}}{\Delta t} = \frac{\Delta \psi}{\Delta t} \hat{k}_E + \frac{\Delta \theta}{\Delta t} \hat{j}_1 + \frac{\Delta \phi}{\Delta t} \hat{i}_b$$

$$\bar{\omega}_b = [p, q, r]^T$$

(body fixed)

$$\bar{\omega}_E = [\dot{\psi}, \dot{\theta}, \dot{\phi}]^T$$

(Earth fixed)

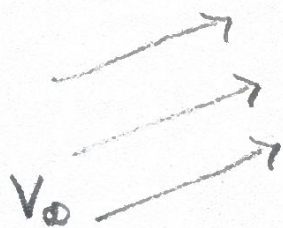
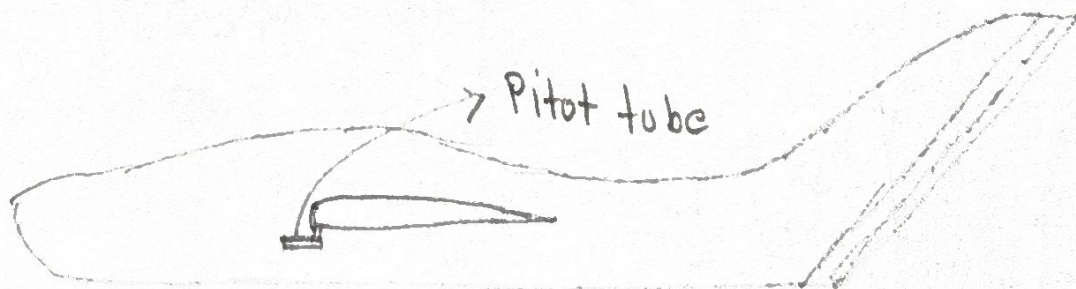
$$\Rightarrow \Delta \dot{\sigma} = \underbrace{\dot{\psi} R_{\psi}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_E}_1 + \underbrace{\dot{\theta} R_{\psi}^T R_{\theta}^T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_1}_2 + \underbrace{\dot{\phi} R_{\psi}^T R_{\theta}^T R_{\phi}^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_2}_3$$

→ Angular rotation (body frame to 3 Coordinate System)

$$\Rightarrow \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan \theta \sin \phi & \tan \theta \cos \phi \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sec \theta \sin \phi & \sec \theta \cos \phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Completely unstable When
 θ approaches to 90°

Problem:- We need to find the actual free Stream Velocity for the given aircraft.



So, that we will have sensors which will find u, v, w
 So that,

$$V_{\infty} = \sqrt{u^2 + v^2 + w^2}$$

$$\alpha = \text{---}$$

$$\beta = \text{--- for body frame}$$

$$u = V_{\infty} \cos \alpha \cos \beta$$

$$v = V_{\infty} \sin \beta$$

{ So, that we can obtain α & β }

$$\Rightarrow \vec{V} = \begin{bmatrix} V_\infty \\ 0 \\ 0 \end{bmatrix}, \quad \bar{\omega}_w = \begin{bmatrix} p_w \\ q_w \\ r_w \end{bmatrix}$$

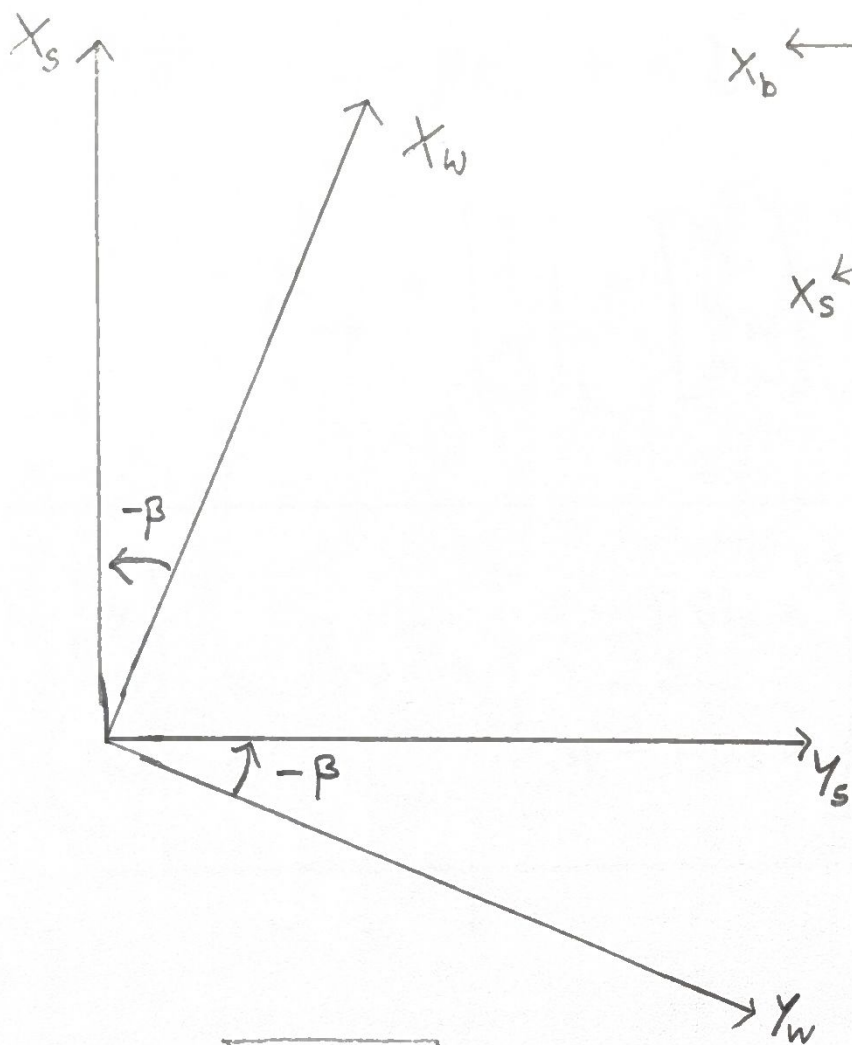
$$\Rightarrow \begin{bmatrix} \hat{l}_E \\ \hat{j}_E \\ \hat{k}_E \end{bmatrix} = R_x^T R_y^T R_u^T \begin{bmatrix} \hat{l}_w \\ \hat{j}_w \\ \hat{k}_w \end{bmatrix}$$

$$\Rightarrow \Delta \dot{\sigma}_w = \underbrace{\dot{x} R_x^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_1 + \underbrace{\dot{y} R_x^T R_y^T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_2 + \underbrace{\dot{u} R_x^T R_y^T R_u^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_3$$

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→ Transformation

$X_w, Y_w, Z_w \xrightarrow{\alpha, \beta} X_b, Y_b, Z_b$ (we have two angles only
So it will not be euler's transformation)



$R_{-\beta}$

$$\Rightarrow \begin{bmatrix} \hat{i}_w \\ \hat{j}_w \\ \hat{k}_w \end{bmatrix} = \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \hat{j}_s \\ \hat{k}_s \end{bmatrix}$$

$R_{-\beta}$

$$\Rightarrow \begin{bmatrix} \hat{i}_s \\ \hat{j}_s \\ \hat{k}_s \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} \hat{i}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \hat{l}_w \\ \hat{j}_w \\ \hat{k}_w \end{bmatrix} = R_E R_\alpha \begin{bmatrix} \hat{l}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix} \quad \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \hat{l}_b \\ \hat{j}_b \\ \hat{k}_b \end{bmatrix} = R_\alpha^T R_E^T \begin{bmatrix} \hat{l}_w \\ \hat{j}_w \\ \hat{k}_w \end{bmatrix}$$

$$\Rightarrow \bar{V}_w = [V_\infty, 0, 0]^T$$

$$\Rightarrow \bar{V}_b = [u, v, w]^T$$

$$\Rightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} V_\infty \cos \alpha \cos \beta \\ V_\infty \sin \beta \\ V_\infty \sin \alpha \cos \beta \end{bmatrix}$$

$$\Rightarrow \vec{F}_w = [-D, Y, -L]^T$$

$$\Rightarrow \vec{F}_b = [F_{Ax}, F_{Ay}, F_{Az}]^T$$

$$\therefore \begin{bmatrix} F_{Ax} \\ F_{Ay} \\ F_{Az} \end{bmatrix} = \begin{bmatrix} \cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \\ \sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha \end{bmatrix} \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix}$$

→ local horizon, wind axes, body axes are attached to the aircraft.

$$\Rightarrow \bar{\omega}_w = [p_w, q_w, r_w]^T \longrightarrow \dot{\chi}, \dot{\psi}, \dot{\mu}$$

$$\Rightarrow \bar{\omega}_b = [p, q, r]^T \longrightarrow \dot{\psi}, \dot{\theta}, \dot{\phi}$$

$$\Rightarrow \Delta \bar{\omega}_b = -\Delta \beta \hat{k}_w + \Delta \alpha \hat{j}_s$$

$$\Rightarrow \dot{\bar{\omega}}_b = -\dot{\beta} \hat{k}_w + \dot{\alpha} \hat{j}_s = \bar{\omega}_b - \bar{\omega}_w$$

$$\therefore \dot{\bar{\omega}}_{wb} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_b - \begin{bmatrix} p_w \\ q_w \\ r_w \end{bmatrix}_w$$

$$\begin{bmatrix} p_w \\ q_w \\ r_w \end{bmatrix} = R_{-\beta} R_{\alpha} \begin{bmatrix} p - \dot{\beta} \sin \alpha \\ q - \dot{\alpha} \\ r - \dot{\beta} \cos \alpha \end{bmatrix} = \begin{bmatrix} (p \cos \alpha + r \sin \alpha) \cos \beta + (q - \dot{\alpha}) \sin \beta \\ -(p \cos \alpha + r \sin \alpha) \sin \beta + (q - \dot{\alpha}) \cos \beta \\ -p \sin \alpha + r \cos \alpha + \dot{\beta} \end{bmatrix}$$

↳ Very important for the first assignment

⇒ It is easy to measure with respect to gravity as our sensors sensing gravity. We are taking equation of motion using C.G. reference for C.G. we will have constant moment of inertia.

Rigid Body Dynamics :-

⇒ Newton's Second law :-

$$F = \frac{d}{dt} (m\bar{v}) ; \bar{M} = \frac{d}{dt} (I\bar{\omega}) \quad [\text{moment}]$$

but this is applicable for inertial reference frame only.

→ for non-inertial reference frame

$$\therefore F = \frac{d}{dt} (m\bar{V}) + \bar{\omega}_b \times m\bar{V}_b$$

$$\bar{\omega}_b = [p, q, r]^T, \bar{V}_b = [u, v, w]$$

→ External forces :-

$$\bar{F} = \bar{F}_{\text{propulsion}} + \bar{F}_{\text{aerodynamics}} + \bar{F}_{\text{gravitational}}$$

We assume that
the propulsive force is acting
along the body frame although
It's not true due to design variation

$$= \begin{bmatrix} F_{Px} \\ F_{Py} \\ F_{Pz} \end{bmatrix} + \begin{bmatrix} F_{Ax} \\ F_{Ay} \\ F_{Az} \end{bmatrix} + \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix}$$

Wind axes
to body axes
transformation

gravitational axes system
to body axis transformation

here,

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \begin{bmatrix} -g \sin \theta \\ g \sin \phi \cos \theta \\ g \cos \phi \cos \theta \end{bmatrix}$$

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$$\Rightarrow m \frac{d\bar{v}}{dt} + \bar{\omega} \times m\bar{v} = \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_A \quad \text{--- ①}$$

$$\Rightarrow \bar{v} = [u, v, w]^T ; \bar{\omega} = [p, q, r]^T$$

$$\Rightarrow \bar{\omega} \times m\bar{v} = \text{Skew-Symmetric } (\bar{\omega}) \cdot m\bar{v}$$

$$\text{if } \bar{a} = [a_1, a_2, a_3]$$

$$\Rightarrow \text{Skew } (\bar{a}) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

$$\Rightarrow \bar{\omega} \times m\bar{v} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} mu \\ mv \\ mw \end{bmatrix}$$

$$\Rightarrow \bar{\omega} \times m\bar{v} = m \begin{bmatrix} -rv + qw \\ ru - pw \\ -qu + pv \end{bmatrix}$$

$\Rightarrow$ So, the LHS of equation ① will be,

$$\Rightarrow m \left(\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} qw - rv \\ ru - pw \\ pv - qu \end{bmatrix} \right)$$

⇒

$$\vec{\mu} = \frac{d}{dt} (\bar{I} \vec{\omega}) + \vec{\omega} \times (\bar{I} \vec{\omega})$$

Inertia matrix.

(Note :- Inertia is something that is not accepting change)

⇒

$$\bar{I} = \begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix}$$

$$I_x = \iiint (y^2 + z^2) dm$$

$$I_{xz} = \iiint xz dm$$

$$I_{yz} = \iiint yz dm$$

$$I_{xy} = \iiint xy dm$$

$$I_z = \iiint (x^2 + y^2) dm$$

$$I_y = \iiint (x^2 + z^2) dm$$

$$\Rightarrow \vec{\omega} \times \bar{I} \vec{\omega} = \begin{bmatrix} 0 & r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

⇒

$$= \begin{bmatrix} -I_{xz} pq - (I_y - I_z) qr \\ -I_{xz} (r^2 - p^2) - (I_z - I_x) pr \\ I_{xz} qr - (I_x - I_y) pr \end{bmatrix}$$

⇒

$$\begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \vec{\omega} \times \bar{I} \vec{\omega} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} \begin{matrix} \text{roll} \\ \text{pitch} \\ \text{yaw} \end{matrix}$$

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$$\Rightarrow m \begin{bmatrix} \dot{u} + qw - rv \\ \dot{v} + ru - pw \\ \dot{w} + pv - qu \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} -I_{xz}pq - (I_y - I_z)qr \\ -I_{xz}(r^2 - p^2) - (I_z - I_x)pr \\ I_{xz}qr - (I_x - I_y)pr \end{bmatrix} = \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan \theta \sin \phi & \tan \theta \cos \phi \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sec \theta \sin \phi & \sec \theta \cos \phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \end{bmatrix} = R_\psi^T R_\theta^T R_\phi^T \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

these are the ground Velocities of the aircraft we can easily convert it to $\begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix}$ North East Down

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_\alpha^T R_\beta^T \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix}$$

We know,

$$\Rightarrow \begin{bmatrix} C_{rx} \\ C_{ry} \\ C_{rz} \end{bmatrix} = R_\phi R_\theta R_\psi \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

and we know for wind axes,

$$\Rightarrow \frac{d}{dt} (\bar{V}_w) + (\bar{V}_w \times m \bar{V}_w) = \bar{F}_w$$

$$\Rightarrow \bar{X} = [u, v, w, p, q, r, \phi, \theta, \psi, x_E, y_E, z_E]^T \rightarrow \text{States of the System}$$

$$\Rightarrow \bar{n} = [\delta_e, \delta_T, \delta_n, \delta_a] \rightarrow \text{Controls that we have.}$$

$$\Rightarrow \bar{Y} = [\underbrace{V_\infty, \alpha, \beta}_{\text{air data boom}}, \underbrace{p, q, r, \phi, \theta, \psi}_{\text{Inertial measuring unit}}, \underbrace{V_N, V_E, h, \lambda}_{\text{GPS}}] \rightarrow \text{Observables (Observation feedback)}$$

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$\Rightarrow$ θ perturbation is very small (in radian) so,

$$\cos \theta = 1$$

$$\sin \theta = 0$$

$$\theta = 0.01 \text{ to } 0.5 \text{ rad}$$

$\downarrow$
large θ

$$\Rightarrow \sin(\theta_0 + \theta) = \sin \theta_0 \cos \theta + \cos \theta_0 \sin \theta$$

$$\Rightarrow \sin(\theta_0 + \theta) = \sin \theta_0 + \theta \cos \theta_0$$

Similarly,

$$\Rightarrow \sin(\phi_0 + \phi) = \sin \phi_0 + \phi \cos \phi_0$$

$$\Rightarrow \cos(\theta_0 + \theta) = \cos \theta_0 \cos \theta - \sin \theta_0 \sin \theta$$

$$\Rightarrow = \cos \theta_0 - \theta \sin \theta_0$$

$$\Rightarrow \cos(\phi_0 + \phi) = \cos \phi_0 - \phi \sin \phi_0$$

$\Rightarrow$ as,

$$\hat{u} = u_0 + u$$

$$\hat{v} = v$$

$$\hat{\omega} = \omega_0 + \omega$$

$$\hat{q} = q$$

$$\hat{p} = p$$

$$\hat{\pi} = \pi$$

$$\hat{q}\hat{u} = q u_0 + q u$$

$$\hat{p}\hat{u} = p u_0 + p u$$

$$\hat{\pi}\hat{u} = \pi u_0 + \pi u$$

$$q \omega_0 + q \omega$$

$$p \omega_0 + p \omega$$

$$\pi \omega_0 + \pi \omega$$

$$\Rightarrow m[\dot{u} + q\omega_0] = T_0 + \Delta T + X_0 + \Delta X - mg \sin(\theta_0 + \theta) = \underbrace{[T_0 + X_0 - mg \sin \theta_0]}_{=0} + \Delta T + \Delta X - mg \theta \cos \theta_0$$

$$\Rightarrow m[\dot{v} + ru_0 - p\omega_0] = \Delta Y + mg \cos(\theta_0 + \theta) \sin(\phi_0 + \phi)$$

$$\Rightarrow = \Delta Y + mg \underbrace{[\cos \theta_0 \sin \phi_0]}_{=0} - \phi \cos \theta_0 \cos \phi_0 - \theta \sin \theta_0 \sin \phi_0 - \theta \phi \cos \theta_0 \sin \phi_0$$

$$\Rightarrow m[\dot{\omega} - qu_0] = Z_0 + \Delta Z + mg \cos(\theta_0 + \theta) \cos(\phi_0 + \phi)$$

$$= Z_0 + \Delta Z + mg \underbrace{[\cos \theta_0 \cos \phi_0]}_{=0} - \theta \sin \phi_0 \cos \theta_0 - \phi \cos \phi_0 \sin \theta_0 + \theta \phi \sin \theta_0 \cos \phi_0$$

Remember for C_L vs α , C_D vs C_L Curve - laminar boundary layer is less stable than turbulence boundary layer.

$$\Rightarrow I_x \dot{p} - I_{xz} \dot{r} = \Delta L$$

$$\Rightarrow I_y \dot{q} = \Delta M$$

$$\Rightarrow I_z \dot{r} - I_{xz} \dot{p} = \Delta N$$

here,

$$T_0 + X_0 = mg \sin \theta_0$$

$$\Rightarrow 0 = mg \cos \theta_0 \sin \phi_0 \Rightarrow \phi_0 = 0$$

$$\Rightarrow Z_0 = -mg \cos \theta_0 \cos \phi_0$$

$$\Rightarrow m[\dot{u} + q\omega_0] = -mg \theta \cos \theta_0 + \Delta X + \Delta T$$

$$\Rightarrow m[\dot{v} + ru_0 - p\omega_0] = \Delta Y + mg \phi \cos \theta_0$$

$$\Rightarrow m[\dot{\omega} - qu_0] = \Delta Z - mg \theta \sin \theta_0$$

$$\dot{\phi} = p$$

$$\dot{\theta} = q$$

$$\dot{\psi} = r$$

This will come out

$$\Rightarrow m\dot{u} = \Delta x - mq\omega_0 - mg\theta \cos \theta_0 + \Delta T$$

$$\Rightarrow m\dot{v} = \Delta y + mp\omega_0 - r u_0 + \phi \cos \phi_0$$

$$\Rightarrow m\dot{w} = \Delta z + mq u_0 - mg\theta \sin \theta_0$$

We can convert the whole equation as:-

$$\Rightarrow \dot{\bar{X}} = A\bar{X} + B\bar{u}$$

$$\Rightarrow \begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \Delta L \\ \Delta M \\ \Delta N \end{bmatrix}$$

$$\Rightarrow x = f(v_\infty, \alpha, q, \delta_e)$$

$$\Rightarrow y = f(\beta, p, r, \delta_a, \delta_r)$$

$$\Rightarrow z = f(v_\infty, \alpha, q, \delta_e)$$

Date :- 28/01/2026

Taylor Series :-

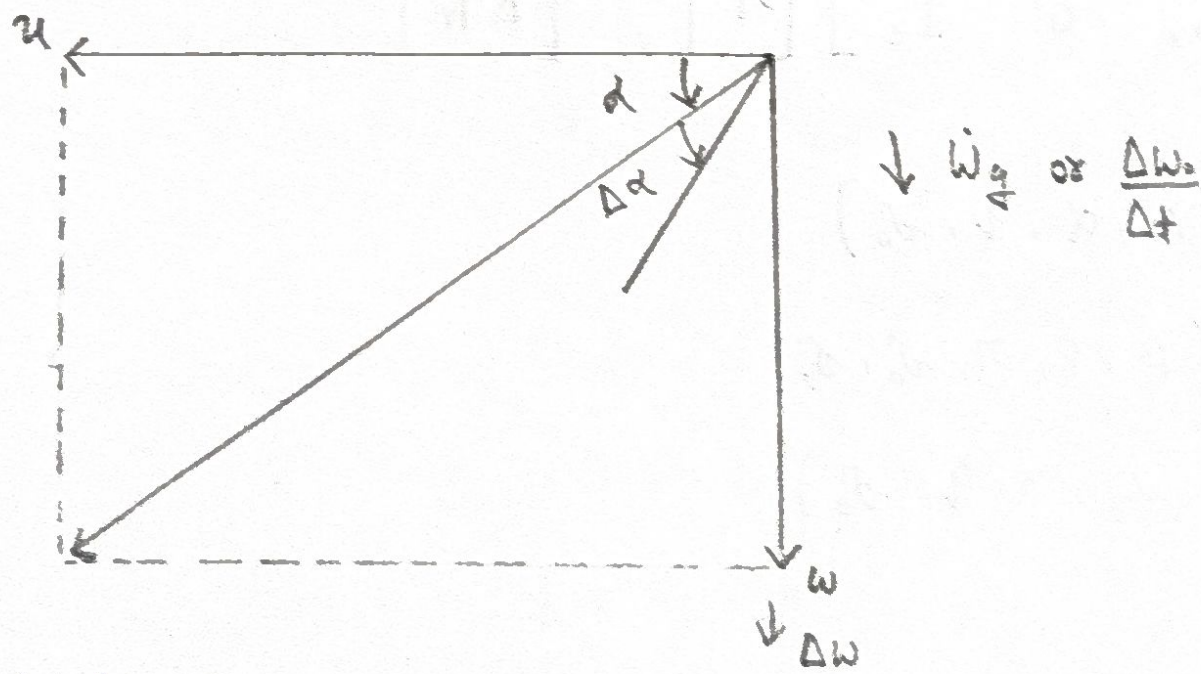
$$\Rightarrow F(x, y) = F_0 + \frac{\partial F}{\partial x} \Delta x + \frac{\partial F}{\partial y} \Delta y + \dots$$

$$\Rightarrow \Delta x = \frac{\partial x}{\partial u} u + \frac{\partial x}{\partial w} w + \frac{\partial x}{\partial q} q + \frac{\partial x}{\partial \delta_e} \delta_e$$

$$\Rightarrow \Delta z = \frac{\partial z}{\partial u} u + \frac{\partial z}{\partial w} w + \frac{\partial z}{\partial \dot{w}} \dot{w} + \frac{\partial z}{\partial q} q + \frac{\partial z}{\partial \delta_e} \delta_e$$

$$\Rightarrow \Delta M = \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \dot{w}} \dot{w} + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \delta_e} \delta_e$$

$$\Rightarrow \Delta T = \frac{\partial T}{\partial \delta_f} \delta_f$$



$\Rightarrow$ list of Variables :-

$$\Delta y = f_1(V, P, \eta, \delta_a, \delta_e)$$

$$\Delta L = f_2(V, P, \eta, \delta_a, \delta_n)$$

$$\Delta N = f_3(V, P, \eta, \delta_a, \delta_n)$$

$\Rightarrow$ We want to form $\dot{\bar{X}} = A\bar{X} + B\bar{u}$

$$\Rightarrow m\ddot{u} = \Delta x + \Delta T - mg\omega_0 - mg\theta \cos\theta_0$$

$$\Rightarrow m\dot{\omega} = \Delta z + mg u_0 - mg\theta \sin\theta_0$$

$$\Rightarrow I_y \ddot{\theta} = \Delta M$$

$$\Rightarrow \dot{\theta} = \dot{q}$$

$$\Rightarrow \ddot{u} = \frac{1}{m} \left[\frac{\partial x}{\partial u} u + \frac{\partial x}{\partial \omega} \omega + \left[\frac{\partial x}{\partial q} - m\omega_0 \right] \dot{q} + \frac{\partial x}{\partial \delta_e} \delta_e \right] - mg\theta \cos\theta_0 + \frac{1}{m} \frac{\partial T}{\partial \delta_T} \cdot \delta_T$$

$$\Rightarrow \dot{\omega} = \frac{1}{m} \left[\frac{\partial z}{\partial u} u + \frac{\partial z}{\partial \omega} \omega + \frac{\partial z}{\partial \dot{\omega}} \dot{\omega} \left[\frac{\partial z}{\partial q} + m u_0 \right] \dot{q} + \frac{\partial z}{\partial \delta_e} \delta_e \right] + g\theta \sin\theta_0$$

$$\Rightarrow \ddot{\theta} = \frac{1}{I_y} \left[\frac{\partial M}{\partial u} u + \frac{\partial M}{\partial \omega} \omega + \frac{\partial M}{\partial \dot{\omega}} \dot{\omega} + \frac{\partial M}{\partial q} \dot{q} + \frac{\partial M}{\partial \delta_e} \delta_e \right]$$

$$\Rightarrow \dot{\theta} = \dot{q}$$

Now we need to find out what is A matrix and B matrix,

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (1 - z\omega) & 0 & 0 \\ 0 & -m\dot{\omega} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \dot{\omega} \\ \ddot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u \\ Z_u \\ M_u \\ 0 \end{bmatrix} \begin{bmatrix} X_\omega & X_q - \omega_0 & -g \cos\theta_0 \\ Z_\omega & Z_q + u_0 & g \sin\theta_0 \\ M_\omega & M_q & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \omega \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \\ 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

$$\Rightarrow X_u = \frac{1}{M} \frac{\partial X}{\partial u} ; \quad M_u = \frac{1}{I_y} \cdot \frac{\partial M}{\partial u}$$

$\Rightarrow$ The solution for $\dot{\bar{X}} = A\bar{X}$ will be $\bar{X}(t) = C e^{\bar{A}t}$

Assignment :- Derive all this equation of motion in longitudinal case only for wind axes system.

$$\left[\frac{1}{M} \frac{\partial \dot{X}}{\partial u} + \rho \left[\frac{\partial M}{\partial u} + \frac{\partial \dot{X}}{\partial \dot{u}} \right] \right] \frac{1}{M} = 0$$

$$\frac{1}{M} \frac{\partial \dot{X}}{\partial u} + \rho \frac{\partial M}{\partial u} = 0$$

$$\left[\frac{1}{M} \frac{\partial \dot{X}}{\partial u} + \rho \left[\frac{\partial M}{\partial u} + \frac{\partial \dot{X}}{\partial \dot{u}} \right] \right] \frac{1}{M} = 0$$

$$\left[\frac{1}{M} \frac{\partial \dot{X}}{\partial u} + \rho \frac{\partial M}{\partial u} + \omega \frac{\partial \dot{X}}{\partial \dot{u}} + \mu \frac{\partial \dot{X}}{\partial \ddot{u}} \right] \frac{1}{M} = 0$$

$$P = 0$$

system of two degrees of freedom of longitudinal motion

$$\begin{bmatrix} \dot{X} \\ \dot{u} \\ \dot{\alpha} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ u \\ \alpha \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{X} \\ \dot{u} \\ \dot{\alpha} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ u \\ \alpha \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Date :- 29/09/2026

$\Rightarrow$ equilibrium states : $V_{00}, \alpha_{trim}, \beta_{trim}, V_{trim}, \phi_{trim}, h_{trim}$

Controls : $\delta e_{tr}, \delta a_{tr}, \delta \pi_{trim}, \delta T_{trim}$

$\Rightarrow X = [\hat{u}, \hat{v}, \hat{\omega}, \hat{p}, \hat{q}, \hat{r}, \hat{\phi}, \hat{\theta}, \hat{\psi}, \hat{x}_E, \hat{y}_E, \hat{z}_E] \rightarrow$ Current State

$$\Rightarrow V = \sqrt{u^2 + v^2 + \omega^2}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{\omega}{u} \right)$$

$$\Rightarrow \beta = \sin^{-1} \left[\frac{v}{V} \right]$$

$$u =$$

$$v =$$

$$\omega =$$

$$\left. \begin{matrix} u = \\ v = \\ \omega = \end{matrix} \right\} f''(v, \alpha, \beta)$$

$$\Rightarrow \Theta = \alpha + \delta$$

$\Rightarrow$ for equilibrium point $\phi \neq 0, \psi \neq 0$

but $p, q, r = 0$

Equilibrium States

$$\rightarrow u_0, v_0, \omega_0, \theta_0, \phi_0 \neq 0$$

$$p_0, q_0, r_0, \psi_0 = 0 \quad \{ \text{all the accelerations are zero} \}$$

$\Rightarrow$ Moment equation for Steady State

$$L_0 = 0$$

$$M_0 = 0$$

$$N_0 = 0$$

→ Force equation for Steady State

$$\rightarrow 0 = -mg \sin \phi_0 + X_0 + T_{x_0}$$

$$\rightarrow 0 = mg \sin \phi_0 \cos \phi_0 + Y_0 + T_{y_0}^{10} \quad \text{[assumed to be zero]}$$

$$\rightarrow 0 = mg \cos \phi_0 \cos \phi_0 + Z_0 + T_{z_0}^{10}$$

So,

$$L_0 = M_0 = N_0 = 0$$

$$X_0 \neq 0, Y_0 \neq 0, Z_0 \neq 0$$

$$\text{if } \beta_0 = 0 \quad \text{then } Y_0 = 0$$

→ Perturbation of States :-

$$\Delta X_p = [u, v, w, p, q, r, \phi, \theta, \psi]^T$$

$$\hat{X} + \Delta \bar{X}_p = \bar{X} = [u_0 + u, v, w_0 + w, p, q, r, \phi_0 + \phi, \theta_0 + \theta, \psi]^T$$

for,

$$V_\infty = 100 \text{ m/s}, \alpha = 2^\circ, u_0 = 99.93 \text{ m/s}, w_0 = 3.48 \text{ m/s}, u_0 \approx V_\infty$$

$$\dot{u}_0 = 0, \dot{w}_0 = 0, \dot{\phi}_0 = 0, \dot{\theta}_0 = 0$$

$$m [\dot{u} + q \hat{w} - r \hat{v}] \rightarrow m [\dot{u} + q w - r u]$$

$$\rightarrow \sin \hat{\theta} = \sin(\theta_0 + \theta)$$

$$\cos \hat{\theta} = \cos(\theta_0 + \theta)$$

$$\hat{x} = x + x_0$$

$$\hat{z} = z_0 + z$$

$$\hat{y} = y$$

$$\hat{T} = T_0 + T$$

Date :- 02/02/2026

Perturbations-

⇒ (a) derivatives w.r.t 'u'

$$\Rightarrow X_u = \frac{1}{m} \frac{\partial X}{\partial u}$$

dimensional

$$\Rightarrow X_u = \frac{1}{m} \left[\frac{1}{2} \rho C_x \frac{\partial V^2}{\partial x} + \frac{1}{2} \rho V^2 S \frac{\partial C_x}{\partial u} \right]$$

$$\Rightarrow X_u = \frac{1}{m} \left[\frac{1}{2} \rho S C_x 2u_0 + \frac{1}{2} \rho u_0 S \frac{\partial C_x}{\partial u} \right]$$

$$\Rightarrow X_u = \frac{1}{m} \left[\rho u_0 S C_{Tx} + \frac{1}{2} \rho u_0^2 S \frac{\partial C_x}{\partial u} \right]$$

$$\Rightarrow X_u = \frac{1}{2} \frac{\rho S}{m} \left[2u_0 C_{Tx} + u_0^2 \frac{\partial C_x}{\partial u} \right]$$

S_0

$$\Rightarrow C_x = -C_D \cos \alpha + C_L \sin \alpha$$

$$\Rightarrow C_x \approx -C_D + C_L \alpha \approx \boxed{-C_D}$$

larger

$$\Rightarrow C_{D+} = C_{D_0} + K C_{L+}^2$$

$$\Rightarrow u_0^2 \frac{\partial C_x}{\partial u} = -u_0 \frac{\partial C_D}{\partial (u/u_0)}$$

$$\Rightarrow \frac{u_0^2 \partial C_x}{\partial u} = -u_0 C_{Du}$$

$$X = \frac{1}{2} \rho V^2 S C_x$$

$$V^2 = (u_0 + u)^2 + w^2$$

$$= u_0^2 + u^2 + 2u_0 u + w^2$$

$$\frac{\partial V^2}{\partial u} = 2u + 2u_0$$

$$= 2[u_0 + u]$$

$$\boxed{\frac{\partial V^2}{\partial u} \approx 2u_0}$$

$$\boxed{u_0 \gg u, v, w}$$

$$\Rightarrow X_u = \frac{1}{2} \frac{\rho S}{m} [-2u_0 C_{D+} - u_0 C_{Du}]$$

$$\Rightarrow \boxed{X_u = -\frac{\rho S}{m u_0} [2 C_{D+} + C_{D,u}]}$$

$$\Rightarrow \bar{Q}_0 = \frac{1}{2} \rho u_0^2$$

$$\Rightarrow C_{Du} = \frac{\partial C_D}{\partial (u/u_0)} = u_0 \frac{\partial C_D}{\partial u}$$

$$\Rightarrow C_{Du} = \frac{u_0}{a_0} \frac{\partial C_D}{\partial (u/a_0)} = M_0 \frac{\partial C_D}{\partial M}$$

$$\Rightarrow \boxed{C_{Du} = M_0 C_{Dm}}$$

Similarly:

$$\Rightarrow Z_u = \frac{1}{m} \frac{\partial Z}{\partial u}$$

$$\Rightarrow Z_u = \frac{1}{m} \left[\frac{1}{2} \rho C_z \frac{\partial V^2}{\partial u} + \frac{1}{2} \rho V^2 S \frac{\partial C_z}{\partial u} \right]$$

$$\Rightarrow Z_u = \frac{1}{m} \left[\frac{1}{2} \rho S C_z 2u_0 + \frac{1}{2} \rho u_0^2 S \frac{\partial C_z}{\partial u} \right]$$

$$\Rightarrow Z_u = \frac{1}{m} \left[\rho u_0 S C_{z,t} + \frac{1}{2} \rho u_0^2 S \frac{\partial C_z}{\partial u} \right]$$

$$\Rightarrow Z_u = \frac{1/2 \rho S}{m} \left[2u_0 C_{z,t} + u_0^2 \frac{\partial C_z}{\partial u} \right]$$

$$\Rightarrow Z_u = \frac{1}{2} \frac{\rho S}{m} \left[-2u_0 C_{L,t} - u_0 C_{Lu} \right]$$

$$\Rightarrow \boxed{Z_u = -\frac{\rho_0 S}{m u_0} \left[2 C_{L,t} + C_{Lu} \right]}$$

$$\Rightarrow \bar{O}_0 = \frac{1}{2} \rho u_0^2, \quad C_{L,t} = \frac{\omega \cos \vartheta}{\frac{1}{2} \rho u_0^2 S} = \frac{\omega \cos \vartheta}{\bar{O}_0 S}$$

$$Z = \frac{1}{2} \rho V^2 S C_z$$

$$V^2 = (u_0 + u)^2 + \omega^2$$

$$= u_0^2 + u^2 + 2u_0 u + \omega^2$$

$$\frac{\partial V^2}{\partial u} = 2u + 2u_0$$

$$= 2[u_0 + u]$$

$$\boxed{\frac{\partial V^2}{\partial u} \approx 2u_0}$$

$$\Rightarrow u_0^2 \frac{\partial C_z}{\partial u} = -u_0 \boxed{\frac{\partial C_L}{\partial (u/u_0)}} \rightarrow C_{L,u}$$

$$\Rightarrow u_0^2 \frac{\partial C_z}{\partial u} = -u_0 C_{L,u}$$

$$\Rightarrow C_{L,u} = \frac{\partial C_L}{\partial (u/u_0)} = u_0 \frac{\partial C_L}{\partial u}$$

$$\Rightarrow C_{L,u} = \frac{u_0}{a_0} \cdot \frac{\partial C_L}{\partial (u/u_0)} = M_0 \frac{\partial C_L}{\partial M}$$

$$\Rightarrow \boxed{C_{L,u} = M_0 C_{L,M}}$$

ℓ

$$\Rightarrow C_z = -C_L - C_{D,a}$$

$$\Rightarrow C_{z,t} = -C_{L,t} - C_{D,t} a_t$$

$$\Rightarrow \boxed{C_{z,t} \underline{=} -C_{L,t}}$$

$$4 \quad M_u = \frac{1}{I_y} \frac{\partial u}{\partial u} = \frac{1}{I_y} \frac{\partial}{\partial u} \left[\frac{1}{2} \rho V^2 S \bar{c} C_m \right]$$

$$\Rightarrow M_u = \frac{1}{I_y} \left[\frac{1}{2} \rho S \bar{c} C_{m,t} \frac{\partial V^2}{\partial u} + \frac{1}{2} \rho V^2 S \bar{c} \frac{\partial C_m}{\partial u} \right]$$

$$\Rightarrow M_u = \frac{\bar{\rho}_0 S \bar{c}}{u_0 I_y} M_0 \frac{\partial C_m}{\partial M} \left| \frac{\bar{\rho}_0 S \bar{c}}{u_0 I_y} \frac{\partial C_m}{\partial (u/u_0)} \right|$$

(b) Derivatives w.r.t ' ω ' :-

$$\Rightarrow X_{\omega} = \frac{1}{m} \frac{\partial X}{\partial \omega}$$

$$\Rightarrow X_{\omega} = \frac{1}{m} \left[\frac{1}{2} \rho S C_x \frac{\partial V^2}{\partial \omega} + \frac{1}{2} \rho V^2 S \frac{\partial C_x}{\partial \omega} \right]$$

$$\Rightarrow X_{\omega} = \frac{1}{2} \frac{\rho S}{m} \left[2 u_0 \alpha C_x + u_0^2 \frac{\partial C_x}{\partial \omega} \right]$$

$$\hat{\alpha} = \tan^{-1} \left[\frac{\hat{\omega}}{\hat{u}} \right]$$

$$\alpha \approx \frac{\omega}{u_0}$$

$$\omega = u_0 \alpha$$

$$\Rightarrow \text{So, } V^2 = (u_0 + u)^2 + \omega^2$$

$$\Rightarrow \frac{\partial V^2}{\partial \omega} = 2\omega = 2u_0 \alpha$$

&

$$\Rightarrow C_x \approx -C_D - C_L \alpha$$

$$\Rightarrow u_0^2 \frac{\partial C_x}{\partial \omega} = u_0 \frac{\partial C_x}{\partial (u/u_0)}$$

$$\Rightarrow u_0^2 \frac{\partial C_x}{\partial \omega} = -u_0 \frac{\partial [C_D + C_L \alpha]}{\partial \alpha}$$

$$\Rightarrow u_0^2 \frac{\partial C_x}{\partial \omega} = -u_0 \left[C_{D,\alpha} + \alpha \frac{\partial C_L}{\partial \alpha} + C_{L,t} \right]$$

$$\Rightarrow C_{x,t} = -C_{D,t}$$

$$\Rightarrow C_{x,t} = - \left[C_{D,0} + K C_{L,t}^2 \right]$$

$$\Rightarrow X_{\omega} = \frac{1}{2} \frac{\rho S}{m} \left[2u_0 \alpha [C_{D0} + k C_{L0}^2] + u_0 [C_{D\alpha} + \alpha C_{L\alpha} + C_{L0}] \right]$$

$$\Rightarrow X_{\omega} = - \frac{\bar{Q}_0 S}{m u_0} \left[2\alpha [C_{D0} + k C_{L0}^2 + C_{L\alpha}] + C_{D\alpha} + C_{L0} \right]$$

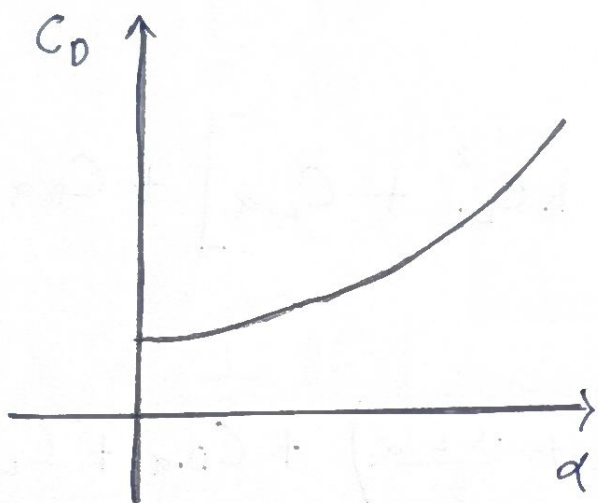
$$\Rightarrow X_{\omega} = - \frac{\bar{Q}_0 S}{m u_0} \left[2\alpha \left(C_{D0} + \frac{C_{L\alpha}}{2} \right) + C_{D\alpha} + C_{L0} \right]$$

$$\Rightarrow X_{\omega} \approx - \frac{\bar{Q}_0 S}{m u_0} [C_{D\alpha} + C_{L0}]$$

$$C_{D0} = C_{D0} + k C_{L0}^2$$

$$, \omega_0 \neq 0, \alpha_0 = \frac{1}{2} \frac{C_{L0}}{C_{L\alpha}}$$

Date :- 03/02/2026



$$C_L = C_{L,0} + C_{L,\alpha}$$

$$C_D = C_{D,0} + k C_L^2$$

$$= C_{D,0} + k \left[C_{L,0}^2 + C_{L,\alpha}^2 \alpha^2 + 2 C_{L,0} C_{L,\alpha} \right]$$

$$\frac{dC_D}{d\alpha} = \frac{2k C_{L,\alpha}^2}{10^{-2} 10^0 10^{-2}} + \frac{2 C_{L,0} C_{L,\alpha}}{10^{-1} 10^0}$$

$$\Rightarrow X_w = \frac{-\bar{Q} S}{m u_0} \left[2 \alpha_0 \left[C_{D,\alpha} + \frac{C_{L,\alpha}}{2} \right] + C_{D,\alpha} + C_{L,\alpha} \right] C_D = C_{D,0} + k_1 C_L + k C_L^2$$

$$\Rightarrow X_w = \frac{-\bar{Q} S}{m u_0} \left[C_{D,\alpha} + C_{L,\alpha} \right]$$

&

$$\Rightarrow Z_w = \frac{1}{m} \frac{dz}{dw}$$

$$\Rightarrow Z_w = \frac{1}{m} \left[\frac{d}{dw} \left(\frac{1}{2} \rho V^2 S C_z \right) \right]$$

$$\Rightarrow Z_w = \frac{1}{2} \frac{\rho S}{m} \left[C_z \frac{dV^2}{dw} + V^2 \frac{dC_z}{dw} \right]$$

$$\Rightarrow Z_w = \frac{1}{2} \frac{\rho S}{m} \left[2 u_0 C_z + u_0 \frac{dC_z}{d\left(\frac{w}{u_0}\right)} \right]$$

$$\Rightarrow Z_w = \frac{1}{2} \frac{\rho u_0 S}{m} \left[2 \alpha_0 \left[-C_L - C_{D,\alpha} \alpha_0 \right] + \frac{dC_z}{d\alpha} \right]$$

$$Z_w =$$

$$\alpha_0 = \frac{w_0}{u_0}$$

$$C_z = -C_L - C_{D,\alpha}$$

$$\Rightarrow \bar{Z}_w = \frac{-\bar{\rho}_0 S}{m u_0} \left[2 x_0 \left[C_L + \frac{C_{D, \alpha}}{2} \right] + C_{L, \alpha} + C_{D, t} \right]$$

$$\Rightarrow \frac{\partial C_z}{\partial \alpha} = -\frac{\partial C_L}{\partial \alpha} - \frac{\partial C_{D, \alpha}}{\partial \alpha} - C_{D, t}$$

$$\Rightarrow = \left[C_{L, \alpha} + C_{D, t} + C_{D, \alpha} \alpha \right]$$

So,

$$\Rightarrow M_w = \frac{1}{I_y} \frac{\partial M}{\partial w}$$

$$\Rightarrow M_w = \frac{\bar{\rho}_0 \bar{S} \bar{c}}{I_y u_0} \frac{\partial C_m}{\partial \alpha}$$

$$\Rightarrow M_w = \frac{\bar{\rho}_0 \bar{S} \bar{c}}{I_y u_0} C_{m, \alpha}$$

(c) derivative w.r.t 'q' :

$$\Rightarrow X_q = \frac{1}{m} \frac{\partial X}{\partial q}$$

$$\Rightarrow Z_q = \frac{1}{m} \frac{\partial Z}{\partial q}$$

$$\Rightarrow M_q = \frac{1}{I_y} \frac{\partial M}{\partial q}$$

$$\Rightarrow X_q = \frac{1}{m} \frac{\partial (\bar{Q}_0 S C_x)}{\partial q}$$

$$\Rightarrow X_q = \frac{\bar{Q}_0 S}{m} \frac{\partial C_x}{\partial q}$$

$$\Rightarrow X_q = \frac{\bar{Q}_0 \bar{S}}{m} \frac{\bar{c}}{2u_0} \boxed{\frac{\partial C_x}{\partial \frac{q\bar{c}}{2u_0}}}$$

$\rightarrow C_{xq}$

$$\Rightarrow \boxed{X_q = \frac{\bar{Q}_0 S \bar{c}}{2m u_0} C_{xq}}$$

$$\Rightarrow n_+ = \frac{Q_+}{Q}, \quad V_H = \frac{S_+ l_+}{S \bar{c}}$$

$$\Rightarrow \Delta L_+ = Q_+ S_+ C_{l_+} \Delta \alpha_+$$

$$\Rightarrow \Delta C_L = n_+ \frac{S_+}{S} C_{l_+} q \frac{l_+}{u_0}$$

$$\Rightarrow = n_+ \frac{S_+}{S} C_{l_+} \frac{l_+}{u_0} \cdot \frac{q\bar{c}}{2u_0} \cdot \frac{2u_0}{\bar{c}}$$

$$\Rightarrow C_{Lq} = \frac{\partial C_L}{\partial \frac{q\bar{c}}{2u_0}} = 2n_+ V_H a_+$$

$$\Rightarrow Z_q = \frac{1}{m} \frac{\partial (\bar{Q}_0 S C_z)}{\partial q}$$

$$\Rightarrow Z_q = \frac{\bar{Q}_0 S}{m} \cdot \frac{\partial C_z}{\partial q}$$

$$\Rightarrow Z_q = \frac{\bar{Q}_0 \bar{S}}{m} \cdot \frac{\bar{c}}{2u_0} \cdot \frac{\partial C_z}{\partial \frac{q\bar{c}}{2u_0}} \cdot C_{zq}$$

$$\Rightarrow \boxed{Z_q = -\frac{\bar{Q}_0 S \bar{c}}{2m u_0} C_{Lq}}$$

$$\left\{ C_{xq} = \frac{\partial C_x}{\partial \frac{q\bar{c}}{2u_0}} \right\}$$

$$\Rightarrow \Delta C_{m, +} = -\Delta L_+ l_+$$

$$\Rightarrow C_{mq} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2u_0}} = -2n_+ V_H l_+ C_{Lq}$$

$$\Rightarrow M_q = \frac{\bar{Q}_0 S \bar{c}}{I_y} \cdot \frac{\partial C_m}{\partial q}$$

$$\Rightarrow \boxed{M_q = -\frac{\bar{Q}_0 S \bar{c}^2}{2u_0 I_y} C_{mq}}$$

Date: 04/02/2026

Derivatives w.r.t. ' $\dot{\omega}$ '

$$\Rightarrow \frac{\dot{\omega}}{u_0} \approx \dot{\alpha}$$

$$\Rightarrow \alpha_t = \alpha + \dot{\alpha} t - \epsilon$$

$$\Rightarrow \alpha_t = \alpha + \dot{\alpha} t - \epsilon_0 - \frac{\partial \epsilon}{\partial \alpha} \alpha - \frac{\partial \epsilon}{\partial \alpha} \cdot \alpha'$$

$$\Rightarrow \alpha_t = \alpha \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) + \dot{\alpha} t - \epsilon_p + \frac{\partial \epsilon}{\partial \alpha} \cdot \alpha \dot{\alpha} t$$

4

$$\Rightarrow \epsilon = \epsilon_0 + \frac{\partial \epsilon}{\partial \alpha} (\alpha - \alpha')$$

$$\Rightarrow \alpha' = \frac{\partial \alpha}{\partial t} \Delta t$$

$$\Rightarrow \alpha' = \alpha' \Delta t$$

$$\Rightarrow \Delta t = \frac{h_t}{u_0}$$

$\dot{\omega}$ need not be external.

$$(a) Z_{\omega} = \frac{1}{m} \cdot \frac{\partial Z}{\partial \dot{\omega}}$$

$$\Rightarrow Z_{\omega} = \frac{1}{m} \frac{\partial}{\partial \dot{\omega}} \left(\frac{1}{2} S V^2 S C_z \right)$$

$$\{ C_z = -C_L - C_{D\alpha} \}$$

$$\Rightarrow Z_{\omega} = - \frac{\bar{Q}_e S}{m} \frac{\partial C_L}{\partial \dot{\omega}}$$

$$\Rightarrow Z_{\omega} = - \frac{\bar{Q}_e S}{m u_0} \cdot \frac{\partial C_L}{\partial \dot{\alpha}}$$

$$Z_{\dot{\omega}} = -\frac{\bar{Q}_0 S}{m u_0} \frac{\bar{C}}{2 u_0}$$

$$\frac{\partial C_L}{\partial \frac{\dot{\alpha} \bar{C}}{2 u_0}}$$

$$\rightarrow C_L \dot{\alpha}$$

$$Z_{\dot{\omega}} = -\frac{\bar{Q}_0 S \bar{C}}{2 m u_0^2} C_L \dot{\alpha}$$

$$Z_{\dot{\omega}} = -\bar{Q}_0 S \bar{C}$$

$$\Delta L_t = Q_t S_t C_{L_t} \Delta \alpha_t$$

$$\Delta C_L = n_t \frac{S_t}{S} C_{L_t} \left[\alpha \left(1 - \frac{\partial \varepsilon}{\partial \alpha} \right) + i_t - \varepsilon_0 + \frac{\dot{\alpha} L_t}{u_0} \right]$$

$$C_{L \dot{\alpha}} = \frac{\partial C_L}{\partial \frac{\dot{\alpha} \bar{C}}{2 u_0}} = 2 n_t V_H C_{L_t} \frac{\partial \varepsilon}{\partial \alpha}$$

$$\Delta M_t = -Q_t S_t C_{L_t} l_t \Delta \alpha_t$$

$$C_{M \dot{\alpha}} = -2 n_t V_H \frac{l_t}{c} C_{L_t} \frac{\partial \varepsilon}{\partial \alpha} = \frac{\partial C_M}{\partial \left(\frac{\dot{\alpha} \bar{C}}{2 u_0} \right)}$$

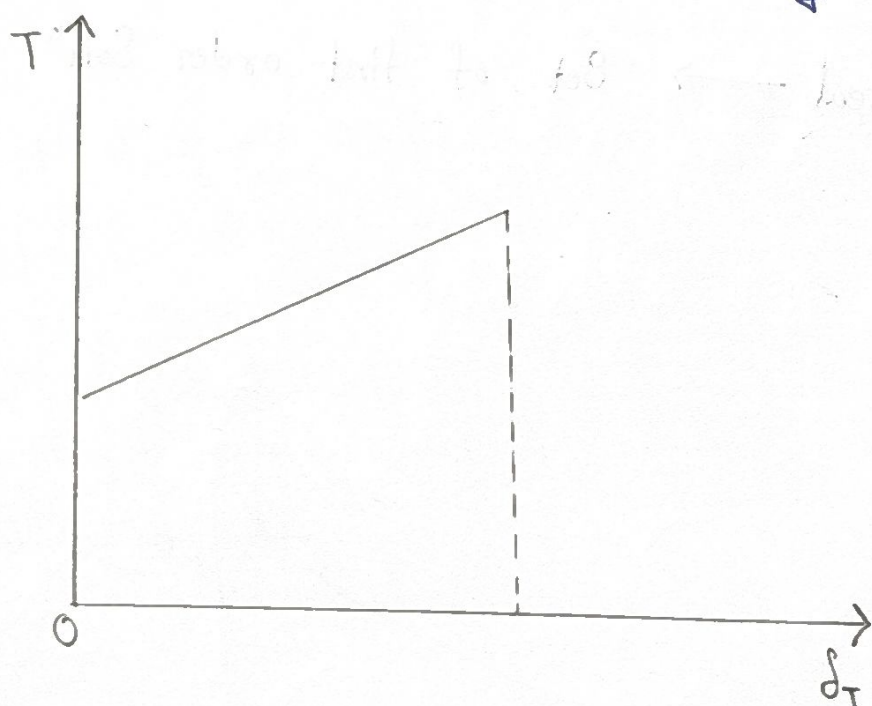
$$M_{\dot{\omega}} = \frac{1}{I_y} \cdot \frac{\partial M}{\partial \dot{\omega}}$$

$$M_{\dot{\omega}} = \frac{\bar{Q}_0 S \bar{C}^2}{2 u_0^2 I_y} \cdot C_{M \dot{\alpha}}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (1-z\dot{\omega}) & 0 & 0 \\ 0 & M\dot{\omega} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\omega} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & (-w_0) & -g\cos\theta_0 \\ Z_u & Z_w & (z_q + v_0) & g\sin\theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \\ 0 \end{bmatrix} \begin{bmatrix} T_{\delta_T} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

$$\Rightarrow X_{\delta_e} = \frac{1}{m} \frac{\partial X}{\partial \delta_e}, \quad Z_{\delta_e} = -\frac{\bar{Q}_0 S}{m u_0} C_{L\delta_e}$$

$$\Rightarrow X_{\delta_e} = -\frac{\bar{Q}_0 S}{m u_0} C_{D\delta_e}, \quad M_{\delta_e} = \frac{\bar{Q}_0 S \bar{c}}{I_y u_0} C_{m\delta_e}$$



$$\dot{\bar{X}} = A \bar{X}$$

$$e^{At} = V e^{\Lambda t} V^+$$

$$AV = V \Lambda$$

Eigenvector matrix

eigenvalue matrix

$$\Rightarrow \begin{bmatrix} \dot{u} \\ \dot{\omega} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} X_u & X_w & -w_0 & -g\cos\theta_0 \\ \frac{Z_u}{1-z\dot{\omega}} & \frac{Z_w}{1-z\dot{\omega}} & \frac{z_q+v_0}{1-z\dot{\omega}} & \frac{g\sin\theta_0}{1-z\dot{\omega}} \\ \left(m_u + \frac{Z_u M_{\dot{\omega}}}{1-z\dot{\omega}}\right) & \left(\frac{M_w + \frac{Z_w M_{\dot{\omega}}}{1-z\dot{\omega}}}{1-z\dot{\omega}}\right) & \left(\frac{M_q + \frac{(z_q+v_0) M_{\dot{\omega}}}{1-z\dot{\omega}}}{1-z\dot{\omega}}\right) & \frac{m_0 g \sin\theta_0}{1-z\dot{\omega}} \\ 0 & 0 & 1 & 0 \end{bmatrix}}_A \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \underbrace{\begin{bmatrix} X_{\delta_e} & T_{\delta_T} \\ \frac{Z_{\delta_e}}{1-z\dot{\omega}} & 0 \\ M_{\delta_e} + \frac{Z_{\delta_e} M_{\dot{\omega}}}{1-z\dot{\omega}} & 0 \\ 0 & 0 \end{bmatrix}}_B \begin{bmatrix} \delta_e \\ \delta_T \end{bmatrix}$$

{each and every Column is linearly independent}

{It allows eigenvalues and eigenvectors to be found easily}

Date: 09/02/2026

$$\dot{\bar{x}} = A \bar{x}$$

$$\dot{\bar{x}} - A \bar{x} = 0$$

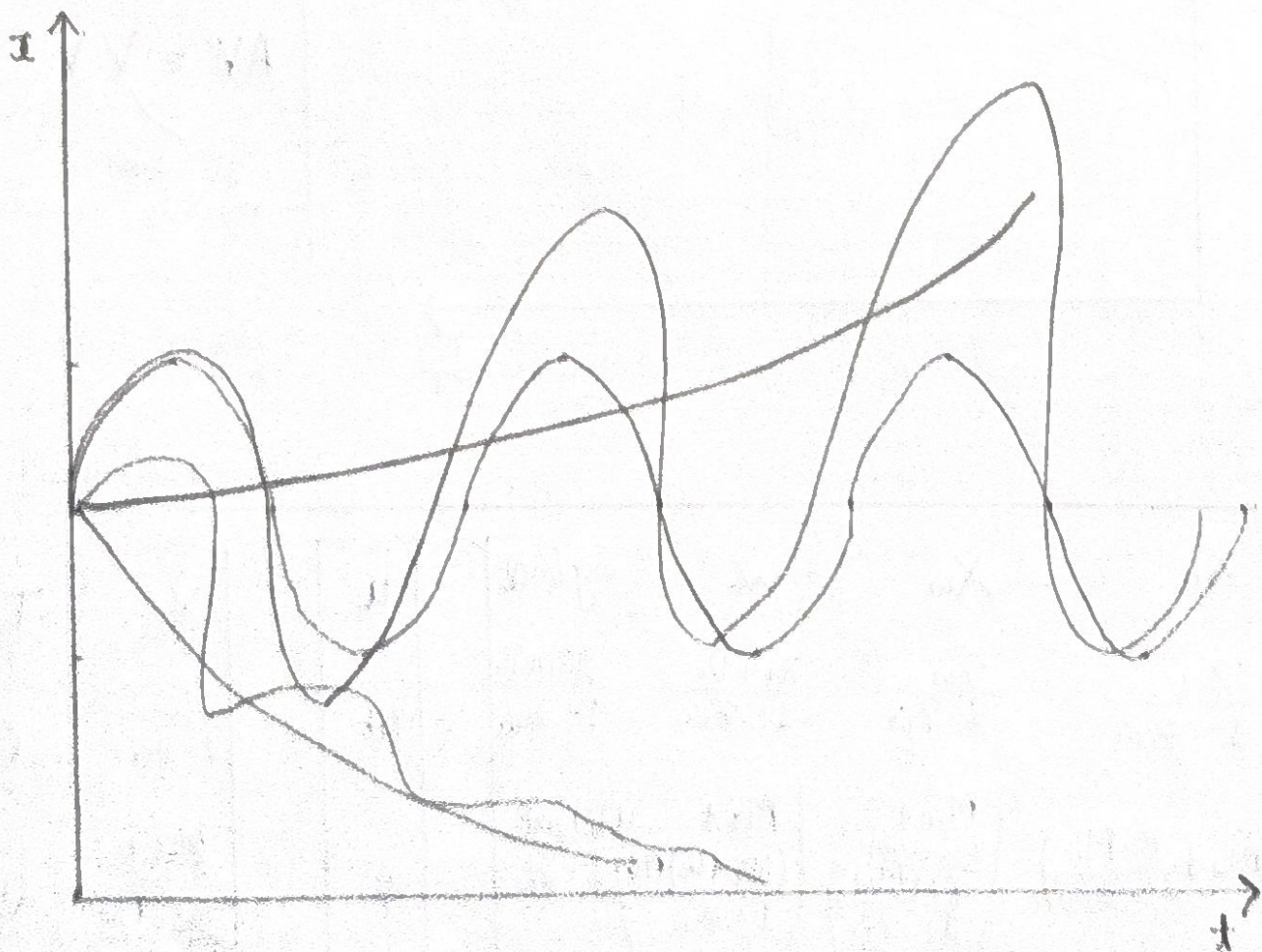
$$\dot{\bar{x}} = C_1 e^{\lambda_1 t}$$

$$\lambda_1 C_1 e^{\lambda_1 t} - A C_1 e^{\lambda_1 t} = 0$$

$$\lambda_1 \bar{x} - A \bar{x} = 0$$

$$A \bar{x} = \lambda_1 \bar{x}$$

$\lambda_i \rightarrow$ distinct & real $\rightarrow$ Set of first order Solⁿ
(repeated)



$\lambda_i \rightarrow$ Complex Conjugate $\rightarrow$ Oscillations

$(\alpha, \pm \beta i) \rightarrow$ Set of 2nd order ODE

Date: 10/02/26

$$\Rightarrow \dot{\bar{x}} = A \bar{x}$$

$$\Rightarrow A \bar{v} = \lambda \bar{v}$$

$\lambda_i \rightarrow$ Real & distinct

$\rightarrow$ Complex Conjugate ($a \pm ib$)

$\Rightarrow a < 0 \rightarrow$ damping

$\Rightarrow a = 0 \rightarrow$ pure oscillatory

$$\Rightarrow a_1 \ddot{x} + a_2 \ddot{x} + a_3 \dot{x} + a_4 x = F(x, \dot{x}, \ddot{x}, t)$$

$$x = C e^{\lambda t}$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{A} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 \end{bmatrix} \begin{bmatrix} x \\ v \\ A \end{bmatrix}$$

$$\dot{x} = v$$

$$\dot{v} = A$$

$$\dot{A} = -a_2 A - a_3 v + a_4 x - F(x, \dot{x}, \ddot{x}, t)$$

Let's Say

$$\Rightarrow \lambda_{1,2,3} = -k_1, -k_2 \pm ik_3$$

then general Solution

$$\Rightarrow x(t) = C_1 e^{k_2 t} + C_2 e^{-k_2 t} [\cos k_3 t \pm i \sin k_3 t]$$

$$\Rightarrow \bar{x}(t) = \bar{C} e^{A t}$$

$$\text{as } e^{at} = I + at + \frac{a^2 t^2}{2!} + \frac{a^3 t^3}{3!} + \dots$$

$$e^{At} = I + At + \frac{A^T A \cdot t^2}{2!} + \dots$$

$$\Rightarrow \Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_n \end{bmatrix}$$

$$\Rightarrow \dot{\bar{x}} = A\bar{x}$$

$$\Rightarrow A\bar{v} = \lambda_i \bar{v}$$

$$\Rightarrow A \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_n \end{bmatrix}$$

$$\Rightarrow AV = V\Lambda$$

$$\Rightarrow \underline{A = V\Lambda V^{-1}}$$

$$\Rightarrow V^{-1}e^{At}V = V^{-1} \left[I + At + \frac{A^T A t^2}{2!} + \dots \right] V$$

$$\Rightarrow = I + \Lambda t + \frac{\Lambda \Lambda t^2}{2!} + \dots$$

$$\Rightarrow V^{-1}e^{At}V = e^{\Lambda t}$$

$$\Rightarrow \underline{e^{At} = V e^{\Lambda t} V^{-1}}$$

Date: 11/02/26

if we have Complex Conjugate roots then how to find ξ

Suppose our equation is

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = F(x, \dot{x}, t)$$
$$x(t) = Ce^{\lambda t}$$

Can be Constant,
Sin, Cos, e, Ramp,
polynomial or anything

$$\Rightarrow \therefore m\lambda^2 + c\lambda + k = 0$$

$$\Rightarrow \therefore \lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0$$

So, typically formulate

$$\Rightarrow \lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$$

$$\Rightarrow \lambda_{1,2} = -\xi\omega_n \pm i\omega_n\sqrt{1-\xi^2}$$

→ A complex eigenvector represent An Oscillatory mode where different State Variables move with phase shift relative to each other

It encodes :- Relative Amplitude
Relative phase difference.

→ Resonance :- Input frequency matches with output

Date: 16/02/26

$$\Rightarrow \begin{bmatrix} \ddot{u} \\ \ddot{w} \\ \ddot{q} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & -\omega_0 & -g \cos \theta_0 \\ Z_{u1} & Z_{w1} & Z_{q1} & Z_{\theta1} \\ M_{u1} & M_{w1} & M_{q1} & M_{\theta1} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}$$

$$\Rightarrow Z_{u1} = \frac{Z_u}{1 - Z_{\dot{w}}} ; \quad Z_{q1} = \frac{u_0 + Z_q}{1 - Z_{\dot{w}}}$$

$$\begin{aligned} \Rightarrow M_{w1} &= M_w + Z_{w1} M_{\dot{u}} \\ \Rightarrow M_{q1} &= M_q + Z_{q1} M_{\dot{u}} \\ \Rightarrow M_{\theta1} &= \frac{-g \sin \theta_0 \dot{u}}{1 - Z_{\dot{w}}} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{aligned} Z_{w1} &= \frac{Z_w}{1 - Z_{\dot{w}}} \\ Z_{\theta} &= \frac{-g \sin \theta_0}{1 - Z_{\dot{w}}} \end{aligned}$$

Short period mode:

$$\Rightarrow \begin{aligned} \omega, q &\neq 0 \\ u, \theta &= 0 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_{w1} & Z_{q1} \\ M_{w1} & M_{q1} \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix}$$

$$\Rightarrow \text{Assumption } \frac{l_{\pm}}{c} \gg \frac{X_{\pm}}{c}$$

Long period mode :

$$\Rightarrow q = 0 ; \quad \dot{q} \neq 0$$

$$\Rightarrow \omega \approx 0 ;$$

$$\Rightarrow \dot{\omega} \approx 0$$

$$\Rightarrow \dot{u} = X_u u + (-g \cos \theta_0) \dot{\theta}$$

$$\Rightarrow 0 = Z_u u + \left(\frac{-g \sin \theta_0}{1 - Z_{\dot{\omega}}} \right) \dot{\theta} + Z_{\dot{\omega}} \dot{\theta}$$

$$\Rightarrow \therefore -Z_{\dot{\omega}} \dot{\theta} = Z_u u + \frac{-g \sin \theta_0}{1 - Z_{\dot{\omega}}} \dot{\theta}$$

$$\Rightarrow \therefore \begin{bmatrix} \dot{u} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \cos \theta_0 \\ \frac{-Z_u}{u_0 + Z_q} & \frac{g \sin \theta_0}{u_0 + Z_q} \end{bmatrix} \begin{bmatrix} u \\ \dot{\theta} \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & u_0 + Z_q \\ M_w & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix}$$

$$\Rightarrow Z_{w_1} = \frac{Z_w}{1 - Z_{w_2}} = Z_w$$

$$\Rightarrow Z_{q_1} = \frac{u_0 + Z_q}{1 + Z_{w_2}}$$

$$\Rightarrow M_q \cong M_{q_1}$$

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow A = \begin{bmatrix} Z_{w_1} & Z_{q_1} \\ M_{w_1} & M_{q_1} \end{bmatrix}$$

$$\Rightarrow |A - \lambda I| = \begin{vmatrix} Z_{w_1} - \lambda & Z_{q_1} \\ M_{w_1} & M_{q_1} - \lambda \end{vmatrix}$$

$$\Rightarrow = (Z_{w_1} - \lambda)(M_{q_1} - \lambda) - Z_{q_1} M_{w_1}$$

$$\Rightarrow = Z_{w_1} M_{q_1} - \lambda Z_{w_1} - \lambda M_{q_1} + \lambda^2 - Z_{q_1} M_{w_1}$$

$$\Rightarrow = \lambda^2 - \lambda(Z_{w_1} + M_{q_1}) + Z_{w_1} M_{q_1} - Z_{q_1} M_{w_1} = 0$$

By Comparing with $\lambda^2 + 2 \zeta \omega_n \lambda + \omega_n^2 = 0$

We will get

$$\omega_n = \sqrt{Z_{w_1} M_{q_1} - M_{w_1} Z_{q_1}}$$

and,

$$\Rightarrow \xi = \frac{-(Z_{w_1} + M_{q_1})}{2 \sqrt{Z_{w_1} M_{q_1} - M_{w_1} Z_{q_1}}}$$

Why we are not considering C_{mq} for ~~using~~ wing?

→ Because

wing moment arm = small

wing induced α from q = small

Tail effects Scales with l_t^2

$$\therefore C_{mq, \text{wing}} \ll C_{mq, \text{tail}}$$

$$\Rightarrow \dot{\alpha} = Z_w \alpha + \frac{u_0 + Z_q}{u_0} q$$

$$\Rightarrow \dot{\alpha} = Z_w \dot{\alpha} + \left(1 + \frac{Z_q}{u_0}\right) q$$

$$\Rightarrow \dot{\alpha} = \frac{Z_\alpha}{u_0} \cdot \dot{\alpha} + \left(1 + \frac{Z_q}{u_0}\right) q$$

$$\Rightarrow Z_\alpha = \frac{1}{m} \frac{\partial Z}{\partial \alpha}$$

$$\Rightarrow Z_w = \frac{1}{m} \frac{\partial Z}{\partial w}$$

$$\Rightarrow Z_w = \frac{1}{m} \cdot \frac{1}{u_0} \cdot \frac{\partial Z}{\partial \alpha}$$

$$\Rightarrow Z_w = \frac{Z_\alpha}{u_0}$$

$$\Rightarrow \dot{Z} = \frac{1}{m} \frac{\partial Z}{\partial \dot{\omega}}$$

$$\Rightarrow \dot{Z} = \frac{1}{m} \cdot \frac{1}{u_0} \cdot \frac{\partial Z}{\partial \dot{\alpha}}$$

$$\Rightarrow \boxed{\dot{Z} = \frac{Z_{\dot{\alpha}}}{u_0}}$$

$$\Rightarrow M_{\omega} = \frac{1}{I_y} \cdot \frac{\partial M}{\partial \dot{\omega}}$$

$$\Rightarrow M_{\omega} = \frac{1}{I_y} \cdot \frac{1}{u_0} \cdot \frac{\partial M}{\partial \dot{\alpha}} = \frac{M_{\dot{\alpha}}}{u_0}$$

$$\Rightarrow \boxed{Z_{\omega_1} = \frac{Z_{\omega}}{1 - Z_{\omega \dot{\alpha}}} \cong Z_{\omega}}$$

$$\Rightarrow Z_{q_1} = \frac{u_0 + Z_q}{1 - Z_{\omega \dot{\alpha}}}$$

$$\Rightarrow M_{\alpha} = u_0 M_{\omega} \quad , \quad M_{\dot{\alpha}} = u_0 M_{\dot{\omega}}$$

$$\Rightarrow Z_{\alpha} = u_0 Z_{\omega} \quad , \quad Z_{\dot{\alpha}} = u_0 Z_{\dot{\omega}}$$

$$\Rightarrow \boxed{M_q \cong M_{q_1}}$$

$$\Rightarrow \begin{bmatrix} \ddot{u} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \cos \theta \\ \frac{-Z_u}{u_0 + Z_q} & \frac{g \sin \theta}{u_0 + Z_q} \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix}$$

for longer Velocities,

$$\Rightarrow \begin{bmatrix} \ddot{u} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \cos \theta \\ -Z_u/u_0 & g \frac{\sin \theta}{u_0} \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} X_u & -g \cos \phi_0 \\ \underbrace{\frac{-Z_u}{u_0 + Z_q}}_{\parallel Z_{u1}} & \underbrace{\frac{g \sin \phi_0}{u_0 + Z_q}}_{\parallel Z_{\phi1}} \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} X_u & -X_\phi \\ Z_{u1} & Z_{\phi1} \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix}$$

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow (X_u - \lambda)(Z_{\phi1} - \lambda) + X_\phi Z_{u1} = 0$$

$$\Rightarrow \therefore X_u Z_{\phi1} - \lambda X_u - \lambda Z_{\phi1} + \lambda^2 + X_\phi Z_{u1} = 0$$

$$\Rightarrow \therefore \lambda^2 - \lambda(Z_{\phi1} + X_u) + X_u Z_{\phi1} + Z_{u1} X_\phi = 0$$

$$\Rightarrow \therefore \omega_n = \sqrt{X_u Z_{\phi1} + Z_{u1} X_\phi}, \quad Z_{u1} = \frac{-Z}{u_0}; \quad Z_{\phi1} = \frac{g \sin \phi_0}{u_0}$$

$$\Rightarrow \therefore \xi = \frac{-Z_{\phi1} + X_u}{\sqrt{X_u Z_{\phi1} + Z_{u1} X_\phi}}$$

$$\frac{1}{u_0 + Z_q} \& 1/u_0 = 10^{-2} \text{ order}$$

$$\phi_0 \text{ small} \Rightarrow Z_{\phi1} = \frac{g \phi_0}{u_0}$$

$$\Rightarrow \lambda^2 - X_u \lambda - \frac{Z_u g}{u_0} = 0$$

assuming $Q_0 = 0$; $u_0 \gg Z_u$

$$\Rightarrow \omega_n = \sqrt{\frac{-Z_u g}{u_0}}$$

$$\Rightarrow \zeta = \frac{X_u}{2 \sqrt{\frac{-Z_u g}{u_0}}}$$

$$\Rightarrow Z_u = -\frac{\bar{Q}_0 S}{m u_0} [2C_{L,trim} + M_0 C_{Lm}]$$

$$\Rightarrow X_u = -\frac{\bar{Q}_0 S}{m u_0} [2C_{D,trim} + M_0 C_{Dm}]$$

$$\Rightarrow Z_0 = \frac{\frac{1}{2} \rho u_0^2 S}{m u_0} \times 2 \times \frac{mg}{\frac{1}{2} \rho u_0^2 S}$$

$$\Rightarrow Z_0 = \frac{-2g}{u_0}$$

$$\Rightarrow X_u = \frac{-\bar{Q}_0 S}{m u_0} \cdot 2C_{L,trim}$$

$$\Rightarrow \omega_n = \sqrt{\frac{-g_0}{u_0} \cdot \frac{-2g}{u_0}}$$

$$\Rightarrow \omega_n = \sqrt{2} \cdot \frac{g}{u_0} \quad (\omega_n \text{ does not depend on the aerodynamic design})$$

$$\Rightarrow \zeta = \frac{-X_u}{2 \sqrt{\frac{-Z_u g}{u_0}}} = \frac{2D_{t,\pi}}{2m u_0} \cdot \frac{1}{\sqrt{2}} \cdot \frac{u_0}{g}$$

Date : 23/02/26

Assignment Solution :-

$$\Rightarrow m\dot{v} = -\Delta D - m g \sin \alpha - \Delta T$$

$$\Rightarrow m v_0 \dot{\alpha} = \Delta L + m v_0 q$$

$$\Rightarrow m v_0 (\dot{\alpha} - q) = \Delta L$$

$$\Rightarrow I_y \dot{q} = \Delta M$$

$$\Rightarrow v = q - \dot{\alpha}$$

$$\Rightarrow m v (\dot{v} = -\Delta L$$

$$\begin{bmatrix} p_w \\ q_w \\ r_w \end{bmatrix} = f_n(\dot{\alpha}, \dot{\beta}, p, q, r)$$

$$\Rightarrow q_w = -(p \cos \alpha + r \sin \alpha) \sin \beta + (q - \dot{\alpha}) \cos \beta$$

$$\Rightarrow q_w = q - \dot{\alpha}$$

$$\Rightarrow \Delta D = \frac{\partial D}{\partial \alpha} \alpha + \frac{\partial D}{\partial v} v + \frac{\partial D}{\partial q} q + \frac{\partial D}{\partial \delta_e} \delta_e$$

$$\Rightarrow \Delta L = \frac{\partial L}{\partial \alpha} \alpha + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial q} q + \frac{\partial L}{\partial \dot{\alpha}} \dot{\alpha} + \frac{\partial L}{\partial \delta_e} \delta_e$$

$$\Rightarrow \Delta M = \frac{\partial M}{\partial \alpha} \alpha + \frac{\partial M}{\partial v} v + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{\partial M}{\partial \delta_e} \delta_e$$

longitudinal Assumptions

$$u_0 = 0 ; u = 0$$

$$\beta_0 = 0 ; \beta = 0$$

$$p = r = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - L \dot{\alpha} & 0 & 0 \\ 0 & -m \dot{\alpha} & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{v} \\ \ddot{\alpha} \\ \ddot{q} \\ \ddot{v} \end{bmatrix} = \begin{bmatrix} D_v & D_\alpha & 0 & -g \\ L_v & L_\alpha & (L_q + 1) & 0 \\ M_v & M_\alpha & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ \alpha \\ q \\ v \end{bmatrix}$$

Aircraft handling qualities

$$\Rightarrow \begin{bmatrix} \dot{\omega} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix}$$

$\lambda_{1,2} \qquad \qquad \lambda_{3,4}$

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = 0$$

$$\Rightarrow x_1 = x$$

$$\Rightarrow x_2 = \dot{x}$$

$$\Rightarrow \dot{x}_2 = -\frac{c}{m}x_2 - \frac{k}{m}x_1$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow x(t) = e^{\lambda t} (\cos(\omega_d t) + \sin(\omega_d t))$$

$$\Rightarrow \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

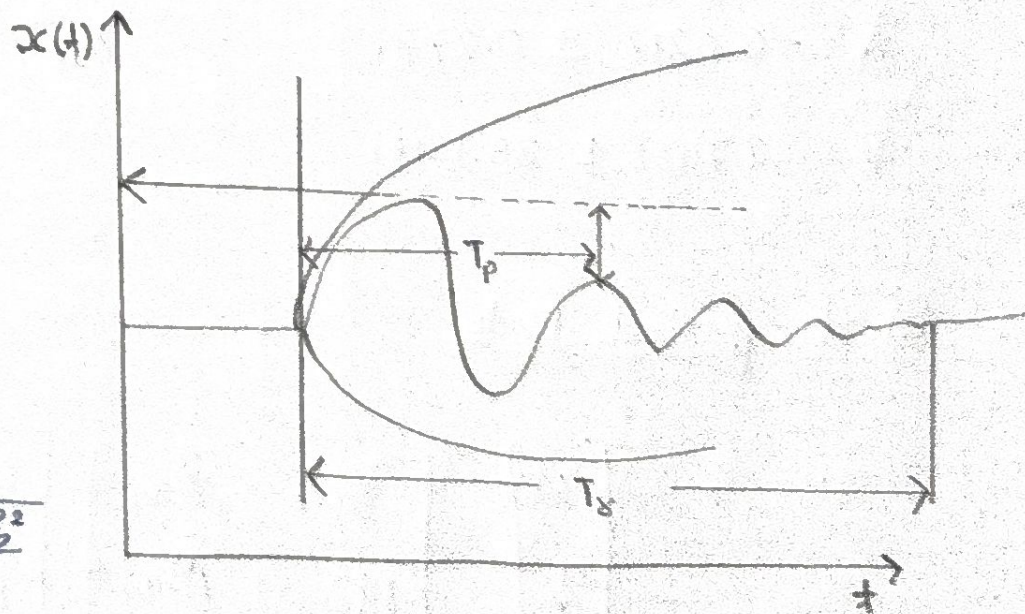
$$\Rightarrow T_p = \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}}$$

Time to

$\frac{1}{n^{th}}$ of amplitude $\frac{\ln(n)}{\omega_n \zeta}$

no. of cycles to $\frac{1}{n^{th}}$ amplitude = $\frac{\ln(n)\sqrt{1-\zeta^2}}{2\pi \zeta}$

for, give all details Calculations



Date: 24/02/26

Phugoid

$$\Rightarrow \begin{bmatrix} \dot{u} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ \frac{-Z_u}{u_0} & \frac{g\theta_0}{u_0} \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix}$$

$$C_{dt} = 0.0426$$

$$\Rightarrow X_u = \frac{-\bar{\Phi}_0 S}{m u_0} (2C_{D,t} + C_{D,u}) = -0.5092 \times [2(0.0426) + 0]$$

$$\Rightarrow X_u = -0.0434$$

$$\Rightarrow Z_u = \frac{-\bar{\Phi}_0 S}{m u_0} (2C_{L,t} + C_{L,u}) = -0.5092 \times 2 \times 0.3853$$

$$\Rightarrow Z_u = -0.3934$$

$$\Rightarrow \begin{bmatrix} \dot{u} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.0434 & -9.81 \\ 0.0079 & 0 \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix}$$

$$\lambda = -0.0217 \pm 0.277i$$

$$\lambda_{sp} = -0.025 \pm 0.297i = \lambda_{3,4}$$

$$V = -2.7468 \pm 35.1316i$$

Short period Mode

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_{w,1} & Z_{q,1} \\ M_{w,1} & M_{q,1} \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix}$$

$$Z_{w1} = \frac{Z_{w0}}{1 - Z_{w0}}, \quad Z_{q1} = \frac{Z_{w0} + Z_{q0}}{1 - Z_{w0}} = \frac{50 + (-8.414)}{1.081} = 39.00$$

$$Z_{w1} = \frac{-2.16}{1.081} = -2.055$$

$$M_{w1} = M_{w0} + Z_{w1} M_{\dot{w}}, \quad M_{q1} = M_{q0} + Z_{q1} M_{\dot{w}}$$

$$M_{w1} = -0.301 - 0.027 \times (-2.055)$$

$$M_{w1} = -0.245$$

$$M_{q1} = -4.74 + (-8.414)(-0.027)$$

$$M_{q1} = -4.513$$

So,

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -2.055 & 39.508 \\ -0.245 & -4.513 \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix}$$

$$\dot{\bar{X}} = A \bar{X}$$

$$|A - \lambda I| = 0$$

$$(-2.055 - \lambda)(-4.513 - \lambda) + 0.245 \times 39.568 = 0$$

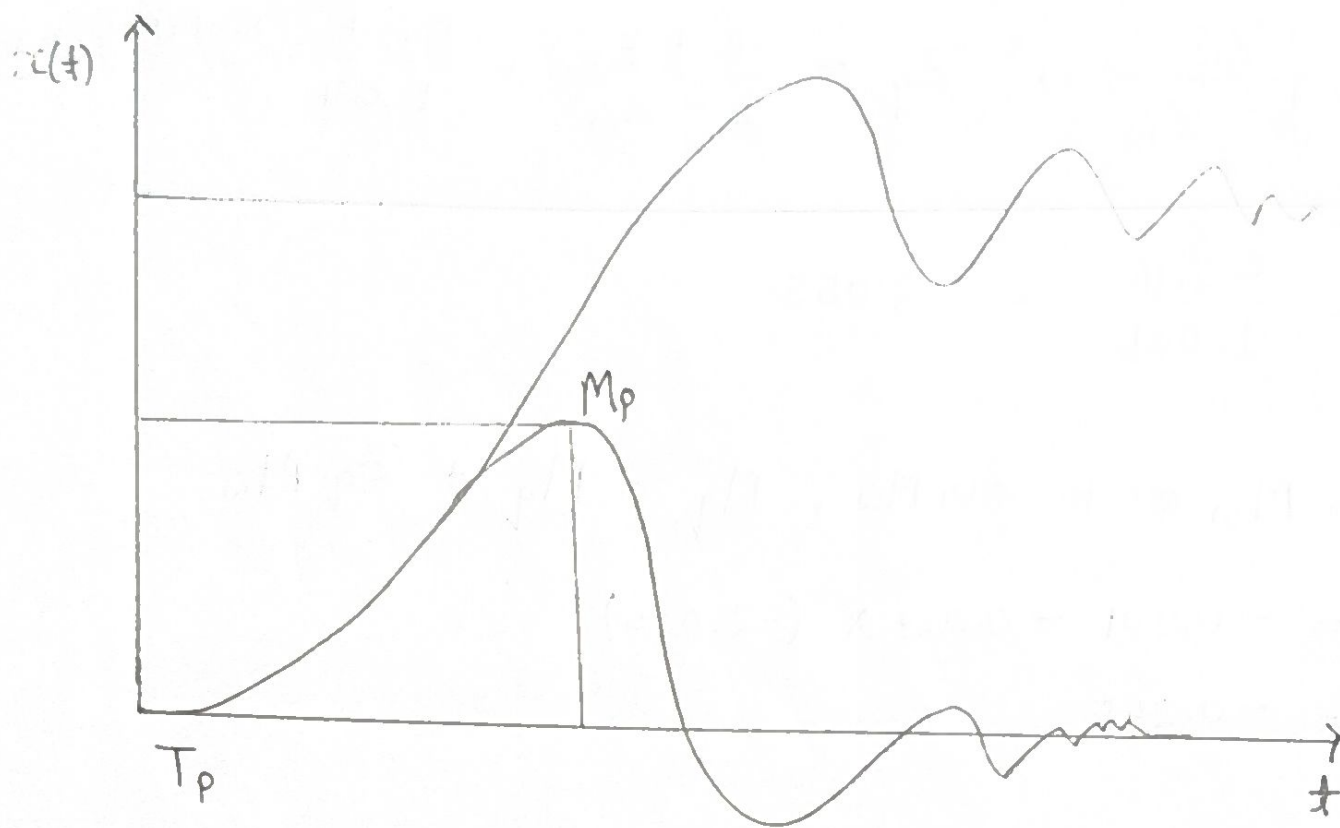
$$\lambda^2 + \lambda(6.568) + 18.968 = 0$$

$$\lambda_{1,2} = \frac{-6.568 \pm \sqrt{-32.733}}{2}$$

$$\lambda_{1,2} = -2.99 \pm 2.98i = \lambda_{sp}$$

$$\lambda_{n,2} = -3.284 \pm 2.861i$$

$$T = 2.11 \text{ s}, \quad T_p = 1.05 \text{ s}, \quad M_p = 4.2\%, \quad T_s = 1.42 \text{ s}$$



for λ_{sp}

$$\Rightarrow \lambda_{1,2} = -2.99 \pm 2.98i$$

$$\Rightarrow T = 2.11s, T_p = 1.05s, M_p = 4.2\%$$

$$\Rightarrow T_s = 1.42s$$

Handling Qualities

Improve the pilot's feedback.

<u>Class</u>	<u>Category</u>	<u>level of acceptance</u>
I $W_{T,0} < 6000 \text{ kg}$	A. Combat, ground Attack Military Exercise	1. decay Adequate (1-3)
II $6000 < W_{T,0} < 30000 \text{ kg}$	B. Climb/Cruise/loiter/ Descent/Refuel/Aerial Delivery	2. Adequate but increase in workload (4-6)
III $W_{T,0} > 30000 \text{ kg}$	C. Takeoff/landing/Power descent/go around.	3. Inadequate to complete mission and safe to Control. (7-10)
IV Fighter, Acrobatic, Missiles		} given by pilots acc. to level

Date : 25/02/26

Lateral Directional

$$\rightarrow u_0 = 0, w_0 = 0, \theta = 0, \cos\theta = 1, \tan\theta = 0$$

$$\rightarrow \left| \begin{array}{l} \phi_0 = 0 ; \psi_0 = 0 \\ p_0 = 0 \end{array} \right| \quad Y_0 + \Delta Y; \Delta h, \Delta N$$

$$\rightarrow m\dot{v} + m[\dot{x}u' - p\omega'] = Y' + T_y + mg \sin\phi' \cos\theta'$$

$$\rightarrow I_x \dot{p}' - I_{xz} \dot{r}' = L'$$

$$\rightarrow I_z \dot{r}' - I_{xz} \dot{p}' = N'$$

$$\rightarrow \therefore \dot{\phi}' = p' + x \tan\theta' \cos\theta'$$

$$\rightarrow \therefore \dot{\psi}' = r' \sec\theta' \cos\phi'$$

$$\rightarrow 0 = Y_0$$

$$\rightarrow 0 = L_0$$

$$\rightarrow 0 = N_0$$

$$\rightarrow m\dot{v} + m\dot{x}u_0 = Y + mg \sin(\phi_0 + \phi) = Y + mg\phi$$

$$\rightarrow \therefore \dot{v} + \dot{x}u_0 = \frac{Y}{m} + g\phi$$

$$\rightarrow I_x \dot{p} - I_{xz} \dot{r} = L$$

$$\rightarrow \therefore \dot{p} = \frac{I_{xz} \dot{r}}{I_x} + \frac{L}{I_x}$$

$$\Rightarrow I_z \dot{\alpha} - I_{xz} \dot{\rho} = N$$

$$\Rightarrow \therefore \dot{\alpha} - \frac{I_{xz}}{I_z} \dot{\rho} = \frac{N}{I_z}$$

$$\Rightarrow \dot{\phi} = p, \dot{\psi} = \pi$$

$$\Rightarrow Y = f_{n1}(v, p, \pi, \delta_a)$$

$$\Rightarrow L = f_{n2}(v, p, \pi, \delta_a, \delta_r)$$

$$\Rightarrow N = f_{n3}(v, p, \pi, \delta_a, \delta_r)$$

$$\Rightarrow \frac{Y}{m} = Y_0 + Y_p p + Y_\pi \pi + Y_{\delta_a} \delta_a$$

$$\Rightarrow Y_0 = \frac{1}{m} \frac{\partial Y}{\partial v}$$

$$\Rightarrow \frac{L}{I_{xx}} = L_v v + L_p p + L_\pi \pi + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r$$

$$\Rightarrow \frac{N}{I_z} = N_v v + N_p p + N_\pi \pi + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{I_{xz}}{I_x} & 0 & 0 \\ 0 & -\frac{I_{xz}}{I_z} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{\pi} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & (Y_\pi - u_0) & 0 & 0 \\ L_v & L_p & L_\pi & 0 & 0 \\ N_v & N_p & N_\pi & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ \pi \\ \phi \\ \psi \end{bmatrix}$$

$$\Rightarrow C_1 = \frac{I_{xz}}{I_x}$$

$$\Rightarrow C_2 = \frac{I_{xz}}{I_z}$$

$$\Rightarrow \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{1-C_1C_2} & \frac{C_1}{1-C_1C_2} & 0 \\ 0 & \frac{C_2}{1-C_1C_2} & \frac{1}{1-C_1C_2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_v & \gamma_p & (\gamma_r - u_0) & g \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

$$\Rightarrow \therefore \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \gamma_v & \gamma_p & (\gamma_r - u_0) & g \\ L_{v_1} & L_{p_1} & L_{r_1} & 0 \\ N_{v_1} & N_{p_1} & N_{r_1} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix}$$

$$\Rightarrow L_{v_1} = \frac{L_v + N_{v_2} C_1}{1 - C_1 C_2}, \quad N_{v_1} = \frac{N_v + L_v C_2}{1 - C_1 C_2}$$

Date :- 24-03-2026

one side smaller dynamic pressure

one side larger dynamic pressure

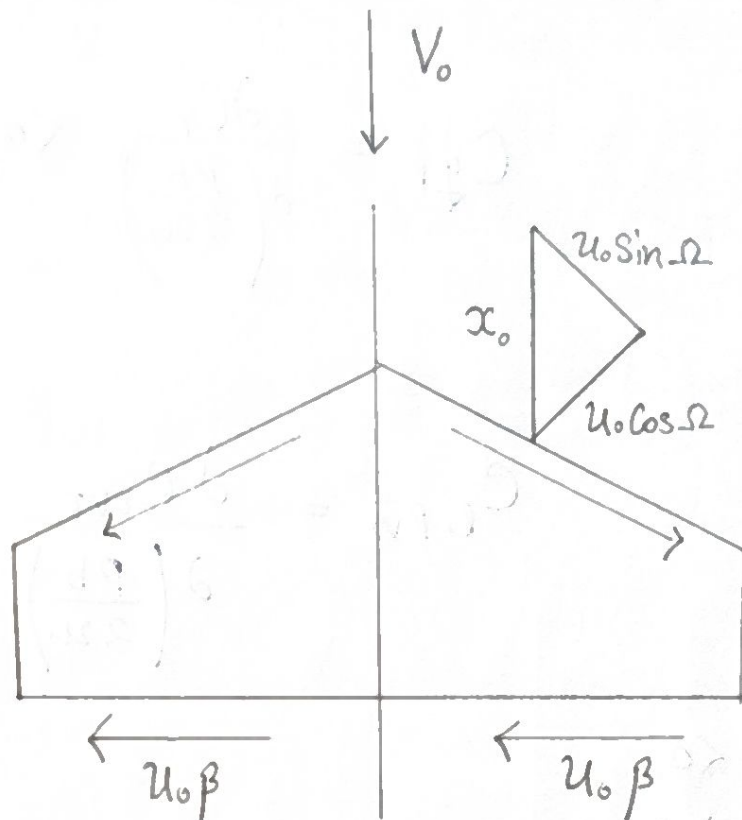
hence, Yaw & Roll moment occurs

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{V_{iv}}{V_c} \right)$$

$$\Rightarrow C_{LBWS} < 0 \text{ or } > 0$$

Wing Sweep

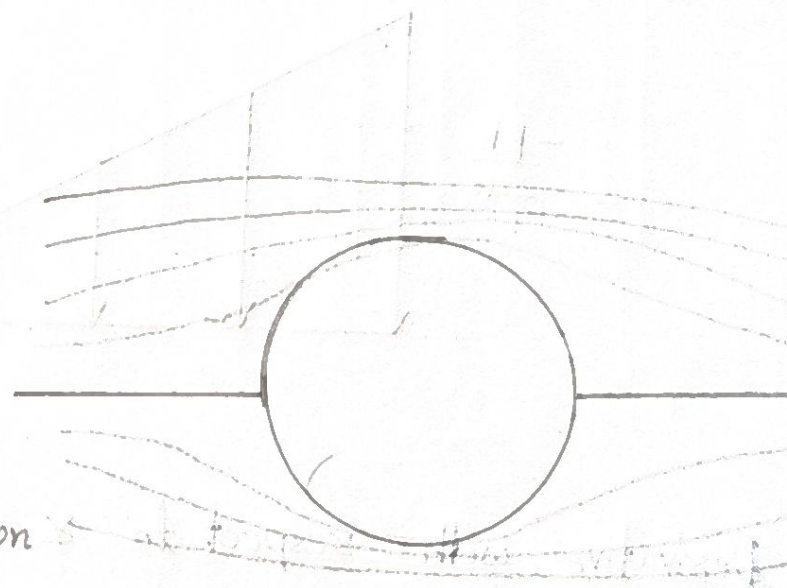
It is wrong



Wing position :-

$\Rightarrow$ low $\rightarrow$ more $C_{LBWS} > 0$

$\rightarrow$ high $\rightarrow C_{LBWS} < 0$
wing location



Derivatives,

w.r.t ρ

$$(a) \quad Y_p = \frac{1}{m} \cdot \frac{\partial Y}{\partial \rho} = \frac{\bar{Q}_0 S}{m} \cdot \frac{\partial C_Y}{\partial \rho} = \frac{\bar{Q}_0 S b}{2 m u_0} < 0$$

$$(b) \quad L_p = \frac{1}{I_x} \cdot \frac{\partial L}{\partial \rho} = \frac{\bar{Q}_0 S b^2}{2 I_x u_0} C_{lp} < 0$$

$$(c) \quad N_p = \frac{1}{I_z} \cdot \frac{\partial N}{\partial \rho} = \frac{\bar{Q}_0 S b}{I_z} \cdot \frac{\partial C_n}{\partial \rho} = \frac{\bar{Q}_0 S b^2}{2 I_z u_0} C_{np} > 0$$

$$\frac{\rho b}{2 u_0}$$

for roll rate and yaw rate

$$C_{Yp} = \frac{\partial C_Y}{\partial \left(\frac{\rho b}{2 u_0} \right)} < 0$$

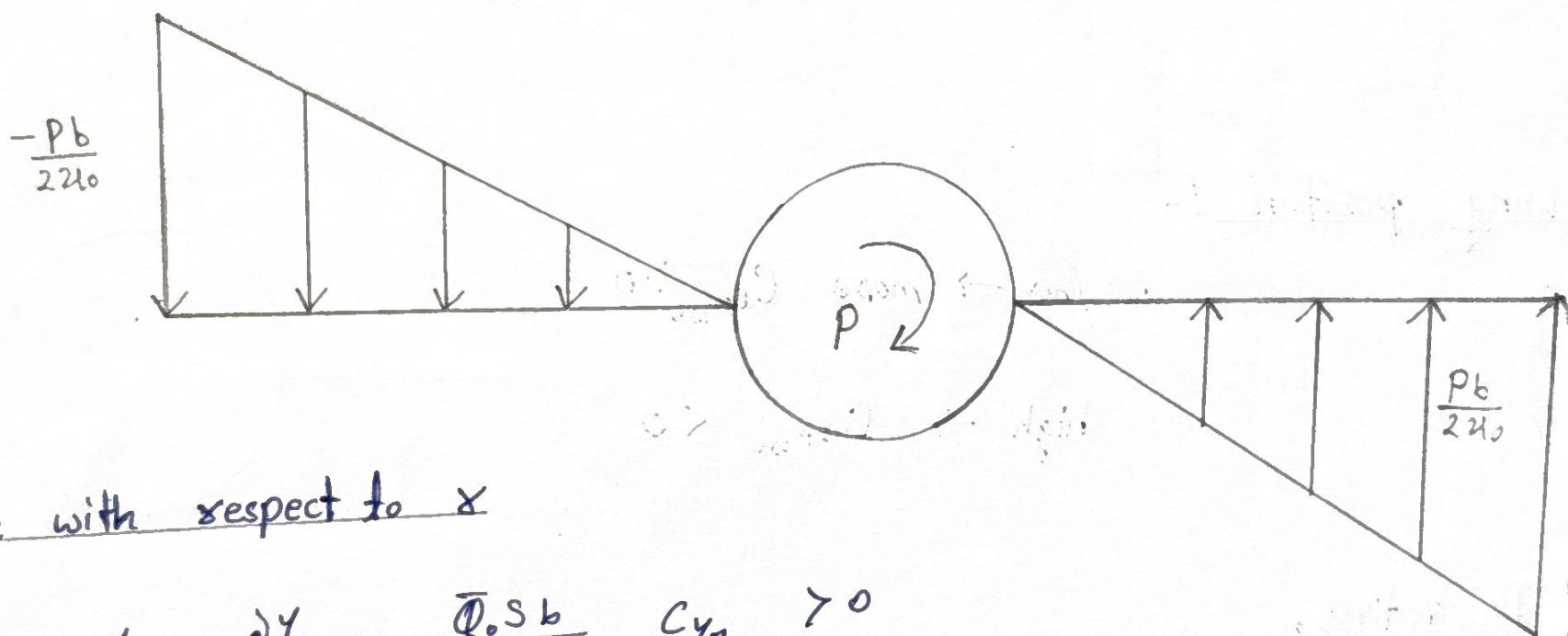
$$C_{NpVT} = \frac{\partial C_{NVT}}{\partial \left(\frac{\rho b}{2 u_0} \right)} > 0$$

$$C_{l.pVT} = \frac{\partial C_{lVT}}{\partial \left(\frac{\rho b}{2 u_0} \right)} < 0$$

{ rolling moment coefficient not lift coefficient }

$$C_{l.pw} < 0$$

$$C_{N.pw} > 0$$



Derivative with respect to α

$$(a) Y_{\alpha} = \frac{1}{m} \cdot \frac{\partial Y}{\partial \alpha} = \frac{\bar{Q}_0 S b}{2 m u_0} C_{Y\alpha} > 0$$

$$(b) L_{\alpha} = \frac{1}{I_x} \cdot \frac{\partial L}{\partial \alpha} = \frac{\bar{Q}_0 S b^2}{2 I_x u_0} C_{L\alpha} > 0$$

$$(c) N_{\alpha} = \frac{1}{I_z} \cdot \frac{\partial N}{\partial \alpha} = \frac{\bar{Q}_0 S b^2}{2 I_z u_0} C_{N\alpha} < 0$$

$$\Rightarrow C_{Y\alpha} > 0, C_{N\alpha VT} < 0, C_{l,\alpha VT} > 0$$

$$\Rightarrow C_{m,\dot{\alpha}} < 0, C_{l,\dot{\alpha}} > 0$$

for aeliron part,

$$N_{sa} = \frac{\overline{Q}_o \cdot S_b}{I_z} C n_{su} > 0$$

Left $\downarrow$ $\oplus$ Right $\uparrow$
acceleration acceleration

Date :- 25/03/2026

$$\Rightarrow C_1 = \frac{I_{xz}}{I_x}, \quad C_2 = \frac{I_{xz}}{I_z}, \quad C_3 = C_1 - C_2$$

$$\Rightarrow L_v = \frac{L_v + N_v C_1}{1 - C_3}; \quad N_v = \frac{N_v + L C_2}{1 - C_3}$$

$$\Rightarrow \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{\pi} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & Y_{\pi} - \dot{u} & g \cos \theta_0 & 0 \\ L_{v2} & L_{p1} & L_{\pi1} & -g \sin \theta_0 & 0 \\ N_{vi} & N_{p1} & N_{\pi1} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ \pi \\ \phi \\ \psi \end{bmatrix} + \begin{bmatrix} Y_{\delta_1} & 0 \\ L_{\delta_{\epsilon1}} & L_{\delta_{a1}} \\ N_{\delta_{\epsilon1}} & N_{\delta_{a1}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{\pi} \\ \delta_u \end{bmatrix}$$

1. Roll mode,

$$\Rightarrow p \neq 0, \quad \dot{p} \neq 0; \quad v = \dot{v} = \pi = \dot{\pi} = 0$$

$$\Rightarrow \dot{p} = L_{p1} p + L_{\delta_u} \delta_u$$

2. Dutch roll mode

⇒

$$V \neq 0, \dot{V} \neq 0; \pi \neq 0; \dot{\pi} \neq 0$$

⇒

$$p = \dot{p} = \phi = \dot{\phi} \approx 0$$

3. Spiral mode

⇒

$$V \neq 0, \phi \neq 0; \pi \neq 0$$

$$\begin{bmatrix} \dot{p} = -r \\ \dot{r} = p \\ \dot{\phi} = 0 \\ \dot{\psi} = 0 \\ \dot{\pi} = 0 \end{bmatrix}$$

limit cycle - limit

$\dot{V}$
 $\dot{\phi}$
 $\dot{\psi}$
 $\dot{\pi}$

limit cycle

$\dot{V}$
 $\dot{\phi}$
 $\dot{\psi}$
 $\dot{\pi}$

Date 1 - 01/04/2026

$$\Rightarrow \dot{X} = Ax + Bu \quad X(t+1) = e^{-At} + \int \tau \cdot (dt)$$

$$\Rightarrow X = f(x, u)$$

$$\Rightarrow X = [u, v, w, p, q, r, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}]$$

$$\Rightarrow \dot{u} = f(\alpha, a_x, g, u)$$

trim condition,

$$\Rightarrow \begin{bmatrix} T = D \\ L = W \\ q = 0 \\ \alpha = 0 \\ \gamma = 0 \end{bmatrix} \begin{array}{l} \rightarrow \text{Validate} \\ \rightarrow \text{for next steps} \end{array}$$

Longitudinal

$$\begin{array}{l} \dot{u} \\ \dot{w} \end{array} \left. \vphantom{\begin{array}{l} \dot{u} \\ \dot{w} \end{array}} \right\} \begin{array}{l} \dot{v} \\ \dot{\alpha} \end{array}$$
$$\begin{array}{l} \dot{\theta} \\ \dot{\phi} \\ \dot{z} \\ \dot{x} \end{array}$$

Lateral - directional

$$\begin{array}{l} \dot{v} \rightarrow \dot{\beta} \\ \dot{p} \rightarrow \dot{\gamma} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \\ \dot{y} \end{array}$$

$$\Rightarrow F = F_T + F_g + F_A$$

$\rightarrow$ 6 Coefficient (moment) for 6 Dof

$$\Rightarrow C_Y = C_{Y_0} + C_{Y\beta} \beta + C_{Yp} \frac{pb}{2V} + C_{Y\delta} \frac{\delta b}{2V} + C_{Y\delta\pi} \delta\pi$$

$$\Rightarrow C_L = C_{L_0} + C_{L\beta} \beta + C_{Lp} p + C_{L\pi} \pi + C_{L\delta a} \delta a + C_{L\delta\pi} \delta\pi$$

$$\Rightarrow C_n = C_{n_0} + C_{n\beta} \beta + C_{np} \bar{p} + C_{n\pi} \pi + \cancel{C_{n\delta a} \delta a} + C_{n\delta a} \delta a$$

$$\Rightarrow C_L = C_{L_0} + C_{L\alpha} \alpha + C_{Lq} \frac{q\bar{c}}{2V} + C_{\delta e} \delta e$$

$$\Rightarrow C_D = C_{D_0} + \dots, C_{Du} = 0$$

$$\Rightarrow C_m = C_{m_0} + C_{m\alpha} \alpha + C_{m\bar{q}} \bar{q} + C_{m\delta e} \delta e$$

Dutch roll & Spiral mode is important.

$$0 = \left(\frac{dA}{dt} + Y \cdot M - (M \cdot X) + A \cdot (M + Y) \right) \cdot X$$

$$M + Y \cdot M - M \cdot X + A \cdot (M + Y) = 0$$

$$(M + Y)$$

Date: 06/04/2026

Roll Mode

$$\Rightarrow V = \dot{V} = \dot{\eta} = \ddot{\eta} = 0$$

$$\Rightarrow \dot{p} = f_{sp} p \Rightarrow f_{roll} = A_1 e^{\lambda_{p1} t}$$

$$\Rightarrow \ddot{\phi} = p \leftarrow \ddot{\phi} = h_{p1} \dot{\phi}$$

$$\lambda_{p1} = L_{p1} < 0$$

$$T_{p1} = \frac{-1}{L_{p1}}$$

$$\phi = h_{p1} \dot{\phi} = 0$$

Deutch roll

$$\Rightarrow \dot{p} = 0, p = 0; \dot{\phi} = \ddot{\phi} = 0$$

$$\Rightarrow \begin{bmatrix} \dot{V} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} Y_v & Y_{\eta} - u_0 \\ N_{v1} & N_{\eta1} \end{bmatrix} \begin{bmatrix} V \\ \eta \end{bmatrix}$$

$$\Rightarrow \lambda^2 - (Y_v + N_{\eta1}) \lambda + (Y_v N_{\eta1} - N_{v1} Y_{\eta} + \frac{N_{v2}}{u_0}) = 0$$

$$\Rightarrow \omega_{n_{D\eta}} = \sqrt{\frac{Y_{\eta} u_0 N_{\eta1} - N_{v1} Y_{\eta} u_0 + N_{v2}}{u_0}}$$

$$\Rightarrow T_n = \frac{-1}{2 \omega_{n_{D\eta}}} (Y_v + N_{\eta1})$$

$$\Rightarrow N_{v1} = \frac{(C_{np})}{N_v} + \frac{(C_{np})}{1 - C_3} \frac{C_2 I_v}{1 - C_3}$$

always
+ve

< 1

Spiral

$$\Rightarrow \dot{\phi} = 0 ; \dot{p} \approx 0, p \approx 0, \dot{\phi} = 0, \dot{V} = 0, V = 0, \dot{\pi} \neq 0, \pi \neq 0$$

$$\Rightarrow \dot{\pi} = N_{v2} V + N_{\pi1} \pi$$

$$\Rightarrow \dot{p} \approx 0 = L_{v1} V + L_{\pi1} \pi$$

$$\Rightarrow V = \frac{-L_{\pi1}}{L_{v1}} \pi$$

$$\Rightarrow \dot{\pi} = \left[N_{\pi1} - N_{v1} \frac{L_{\pi1}}{L_{v1}} \right] \pi$$

$$\Rightarrow \dot{\pi} = N_{sp} \pi$$

$$\Rightarrow \pi(t) = A_1 e^{\lambda_{sp} t}$$

for Stable Spiral mode

$$\Rightarrow \lambda_{sp} < 0$$

$$\Rightarrow N_{\pi1} - N_{v2} \cdot \frac{L_{\pi1}}{L_{v1}} < 0$$

$$\Rightarrow \frac{L_{v1} N_{\pi1} - N_{v1} L_{\pi1}}{L_{v1}} < 0$$

$$\Rightarrow \frac{\overset{(-ve)}{L_v} \overset{(-ve)}{N_{\pi}} - \overset{(+ve)}{N_v} \overset{(+ve)}{L_{\pi}}}{\underset{(-ve)}{L_v}}$$

$$\frac{L_v N_{\pi}}{K_1 C_{Lp}} > \frac{L_v L_{\pi}}{K_2 C_{np}}$$

Assume -

$$C_1, C_2 \approx 0$$

$$C_3 \approx 0$$

Clancy, Nelson for interview prep

aerodynamics

Stability

Date: 07/04/2026

Control:-

$$\Rightarrow \bar{X} = f(\bar{X}, \bar{n}, t)$$

$$\Rightarrow \bar{X} = [\bar{v}, \bar{\omega}, \bar{\theta}, \bar{s}]$$

$$\Rightarrow \bar{n} = [\delta_T, \delta_e, \delta_u, \delta_r]$$

desired State Vector is given $\bar{X}_d$
for steady level flight:

$$\Rightarrow \bar{X}_d = [h, V_\infty, \gamma] \Rightarrow \bar{n} = [\delta_T, \delta_e, \delta_r]$$

$$\Rightarrow T = D(V_\infty, h)$$

$$\Rightarrow L = W$$

$$\Rightarrow \gamma = 0$$

measurement Vector

$$\Rightarrow \bar{y} = f(\bar{X}, \bar{v})$$

Instruments

Aircraft Sensor

$$\left. \begin{array}{l} \rightarrow \text{Pitot Static } (V_{x_{\infty}}) \\ \rightarrow \text{AOA Vane } (\alpha) \\ \rightarrow \text{AOSS Vane } (\beta) \end{array} \right\} \begin{array}{l} u \\ \alpha \\ \beta \end{array} \Rightarrow (V_\infty, V, W)$$

Inertial Measuring Unit (IMU)

$$\rightarrow \bar{a} = [a_x, a_y, a_z]$$

$$\rightarrow \bar{\omega} = [p, q, r]$$

$$\rightarrow \bar{\theta} = [\phi, \theta, \psi]$$

$$\rightarrow [\psi, \chi] \rightarrow \text{by magnetometer}$$

Kalman - filter

$\rightarrow$ GPS

$$\begin{aligned} &\rightarrow \text{Longitude, latitude, altitude} \\ &\rightarrow X, Y, h \\ &\rightarrow \bar{V}_n, x, V_h \end{aligned}$$

Convert to

$$\Rightarrow V_G = [V_N, V_E, V_h]$$

$$\Rightarrow \bar{Y} = [V_\omega, \alpha, \beta, p, q, r, \phi, \theta, \psi, x, y, h]$$

$$\Rightarrow \bar{Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2u_\omega & 0 \\ 0 & 0 & 1/2u_\omega \end{bmatrix} \begin{bmatrix} u \\ 0 \\ \omega \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \\ \phi \\ \theta \\ \psi \\ x \\ y \\ h \end{bmatrix}$$

Date : 08/04/2026

Geometry

$$\begin{aligned} \Rightarrow V_0 &= 50 \\ \Rightarrow S &= 12.47 \text{ m} \\ \Rightarrow b &= 10.47 \text{ m} \\ \Rightarrow m &= 750 \text{ kg} \end{aligned}$$

$$I_{xx} = 873 \text{ kg m}^2$$

$$I_{yy} = 907 \text{ kg m}^2$$

$$I_{zz} = 1680 \text{ kg m}^2$$

$$I_{xz} = 1144 \text{ kg m}^2$$

$$\rho = 1.225 \text{ kg/m}^3$$

$$g_0 = 9.81 \text{ m/s}^2$$

$$\Rightarrow \Gamma = I_x I_z - I_{xz}^2$$

$$= 157,904$$

$$\Rightarrow A = \begin{bmatrix} \frac{Y_m}{V} & \frac{I_z L_v + I_{xz} N_v}{\Gamma} & \frac{I_{xz} L_v + I_x N_v}{\Gamma} & 0 \\ \frac{Y_p}{m} & \frac{I_z L_p + I_{xz} N_p}{\Gamma} & \frac{I_{xz} L_p + I_z N_p}{\Gamma} & 1 \\ \frac{Y_n}{m} - V_0 & \frac{I_z L_n + I_{xz} N_n}{\Gamma} & \frac{I_{xz} L_n + I_x N_n}{\Gamma} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow Y_v = k C_{Y\beta}$$

$$\Rightarrow L_v = k b C_{L\beta}$$

$$\Rightarrow N_v = k b C_{N\beta}$$

$$Y_p = \frac{k b^2}{2} C_{Yp}$$

$$L_p = \frac{k b^2}{2} C_{Lp}$$

$$N_p = \frac{k b^2}{2} C_{Np}$$

$$Y_n = \frac{k b}{2} C_{Yn}$$

$$L_n = \frac{k b^2}{2} C_{Ln}$$

$$N_n = \frac{k b^2}{2} C_{Nn}$$

$$\Rightarrow k = \frac{q_s}{V_0}$$

$$\Rightarrow C_{Y\beta} = -0.431$$

$$\Rightarrow C_{Y\beta} = 0.269$$

$$\Rightarrow C_{Y\pi} = 0.433$$

$$\Rightarrow C_{L\beta} = 0.15$$

$$\Rightarrow C_{Lz} = 0.0$$

$$\Rightarrow C_{L\beta} = -0.51$$

$$= -0.041$$

$$\Rightarrow C_{Lp} = -0.851$$

$$= -0.21$$

$$\Rightarrow C_{L\pi} = 0.36$$

$$\Rightarrow C_{L\delta_a} = -0.153$$

$$\Rightarrow C_{L\delta_a} = 0.05$$

$$\Rightarrow C_{n\beta} C_{Lp} - C_{L\beta} C_{n\pi} > 0$$

$$< 0$$

$$C_{n\beta} = 0.71$$

$$= 0.051$$

$$C_{np} = -0.045$$

$$C_{n\pi} = -0.91$$

$$\cancel{C_{n\pi} = 0.91}$$

$$= -0.01$$

$$C_{n\delta_a} = -0.049$$

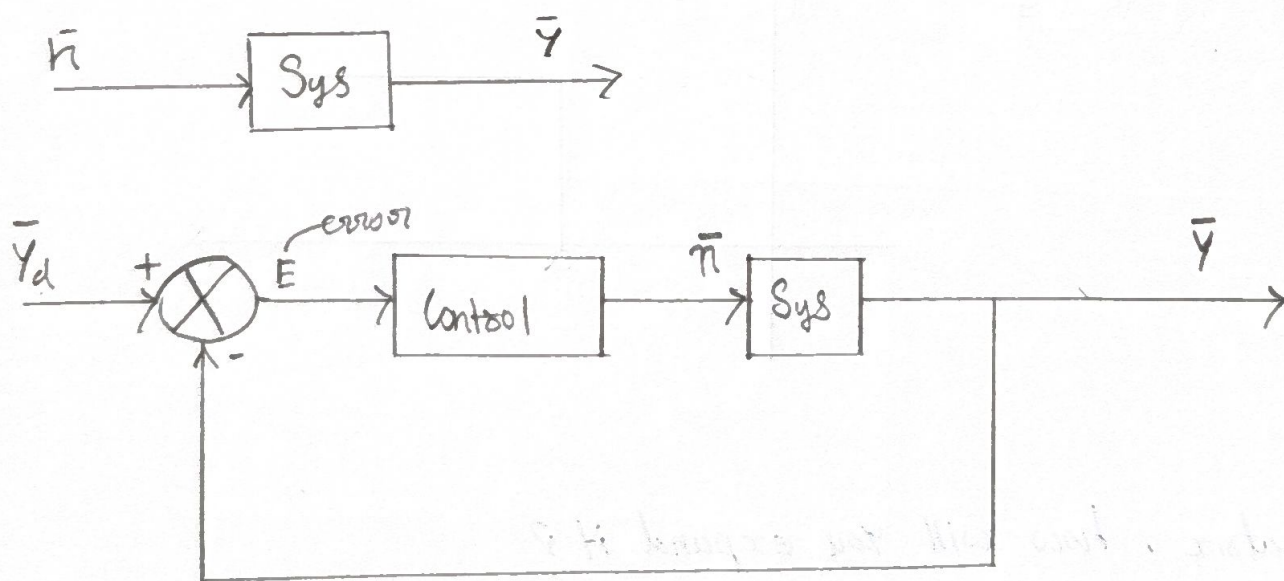
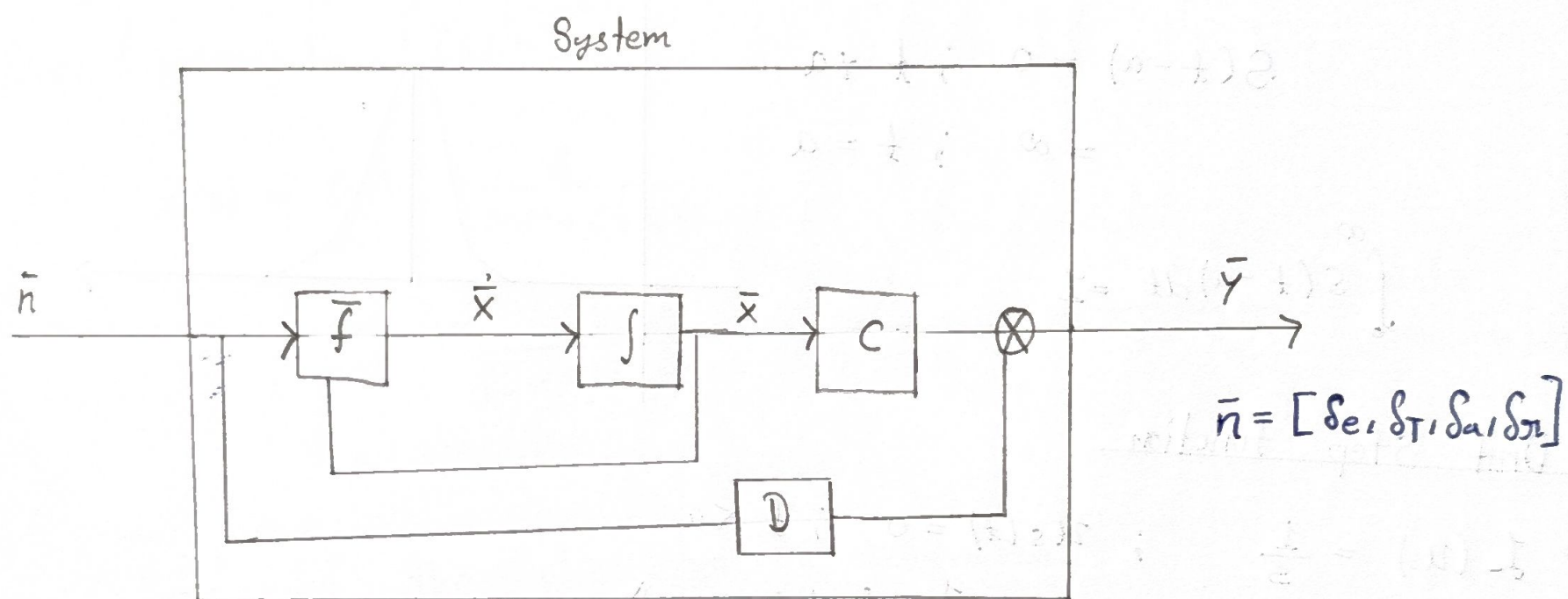
$$C_{n\delta_a} = 0.0$$

Black - another example

Spiral mode flat spin - Youtube.

Date : 13/04/2026

$$\begin{aligned} \Rightarrow \bar{x} &= \bar{f}(\bar{x}, \bar{n}, t) + \bar{w} \\ \Rightarrow \bar{y} &= C\bar{x} + D\bar{n} + \bar{v} \\ \Rightarrow \bar{y} &= \bar{g}(\bar{x}, \bar{n}) \end{aligned} \quad \left. \begin{array}{l} \text{measurement noise} \\ \text{process noise} \end{array} \right\} \Rightarrow \begin{aligned} \dot{\bar{x}} &= A\bar{x} + B\bar{n} \\ \bar{y} &= C\bar{x} + D\bar{n} \end{aligned}$$



Laplace transformation

$$\Rightarrow \mathcal{L} \left[\frac{df}{dt} \right] = sF(s) - f(0)$$

$$\Rightarrow \mathcal{L} [e^{at} f(t)] = F(s-a)$$

$$\Rightarrow \mathcal{L}[f(t-a)] = e^{-as} F(s)$$

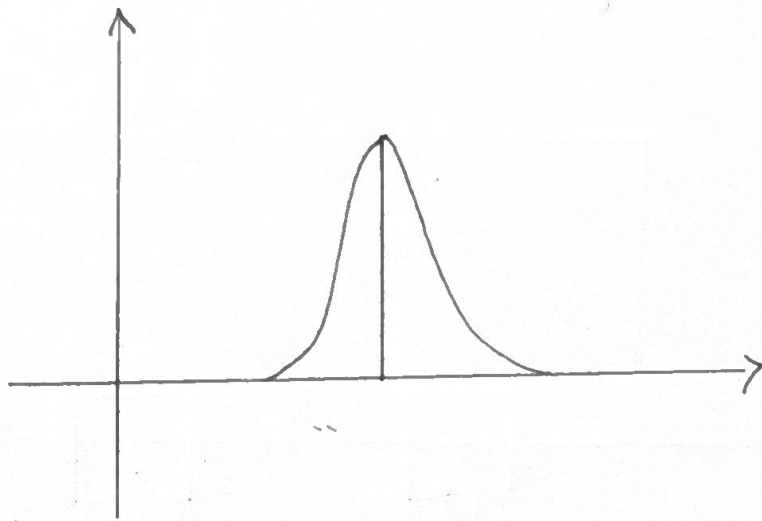
$$\Rightarrow \mathcal{L}[s(t-a)] = 1$$

Unit Impulse function

$$\Rightarrow s(t-a) = 0 \quad ; \quad t \neq a$$

$$\Rightarrow = \infty \quad ; \quad t = a$$

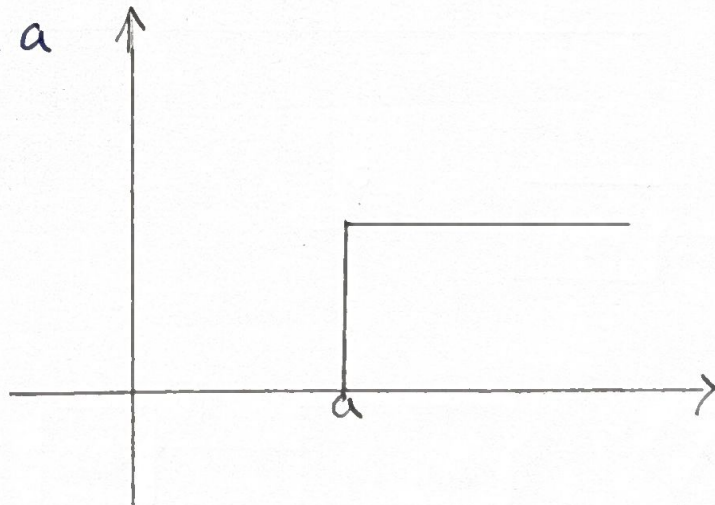
$$\Rightarrow \int_0^{\infty} s(t-a) dt = 1$$



Unit Step function

$$\Rightarrow \mathcal{L}(u_s) = \frac{1}{s} \quad ; \quad u_s(t) = 0 \quad ; \quad t < a$$

$$1 \quad ; \quad t \geq a$$



$\int e^{At} \rightarrow A$ is a matrix. how will you expand it?

$$\Rightarrow m\ddot{x} + (c\dot{x} + kx) = F(t)$$

$$\therefore ms^2 X(s) + c s X(s) + k X(s) - s(\dot{x}_0) - x_0 = F(s)$$

$$\Rightarrow x_0 = 0, \quad \dot{x}_0 = 0$$

$$\Rightarrow X(s) [ms^2 + cs + k] = F(s)$$

State or measurement

$$\therefore X(s) = \frac{F(s)}{ms^2 + cs + k}$$

$$\therefore \boxed{\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}} \rightarrow \text{Transfer function}$$

Impulse function :- $F(s) = 1$

$$\Rightarrow X(s) = G(s) = \frac{1}{ms^2 + cs + k}$$

$$\Rightarrow = \frac{1}{(s+p_1)(s+p_2)} = \frac{A}{s+p_1} + \frac{B}{s+p_2} \quad (\text{Partial function})$$

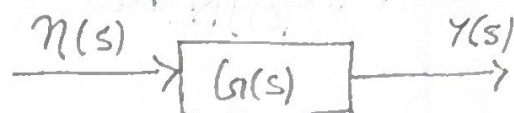
Poles :- Solⁿ of the characteristic polynomial
(roots)

$$\Rightarrow \mathcal{L}^{-1}[\quad] = Ae^{-p_1 t} + Be^{-p_2 t}$$

$$\Rightarrow F(s) = as^3 + bs^2 + cs + d$$

Date :- 17/04/2026

$$\Rightarrow G(s) = \frac{Y(s)}{X(s)} = \frac{N_G(s)}{D_G(s)}$$



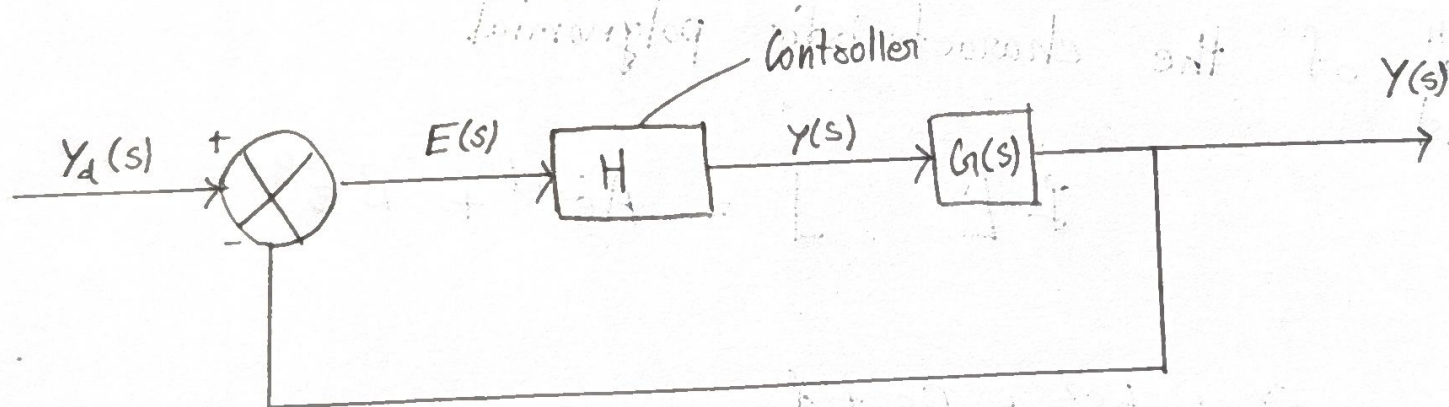
$$\Rightarrow = \frac{\prod_{i=1}^m (s + z_i) (s + z_2) \dots (s + z_j)}{\prod_{i=1}^n (s + p_1) (s + p_2) \dots (s + p_i)} = \text{Zero poles}$$

$n \Rightarrow$ order of characteristic polynomial

Also,

$$\Rightarrow n \geq m \xrightarrow{\text{zero}} \text{proper}$$

$$\Rightarrow n < m \xrightarrow{\text{zero}} \text{Improper}$$



$$\Rightarrow E(s) = Y_d(s) - Y(s)$$

$$\Rightarrow = Y_d(s) - G(s)H(s)E(s)$$

$$\Rightarrow = Y_d(s) - G(s)H(s)E(s)$$

$$\Rightarrow Y_d(s) = [1 + G(s)H(s)]E(s)$$

$$\Rightarrow E(s) = \frac{Y_d(s)}{1 + G(s)H(s)}$$

$$Y(s) = G(s)H(s)E(s)$$

$$E(s) = \frac{Y(s)}{G(s)H(s)} \rightarrow \textcircled{B}$$

$$\boxed{\frac{Y(s)}{Y_d(s)} = \frac{G(s)H(s)}{1 + G(s)H(s)}} \rightarrow G_{cl}$$

Partial fraction

$$\Rightarrow \frac{Y(s)}{F(s)} = \frac{s + z_1}{(s + p_1)(s + p_2)}$$

$$; \frac{1}{(s + p_1)(s + p_2)}$$

$$\Rightarrow \frac{A_1}{s + p_1} + \frac{B}{s + p_2}$$

$$; \frac{A_2}{s + p_1} + \frac{B_2}{s + p_2}$$

Completely different

Some sort of relation

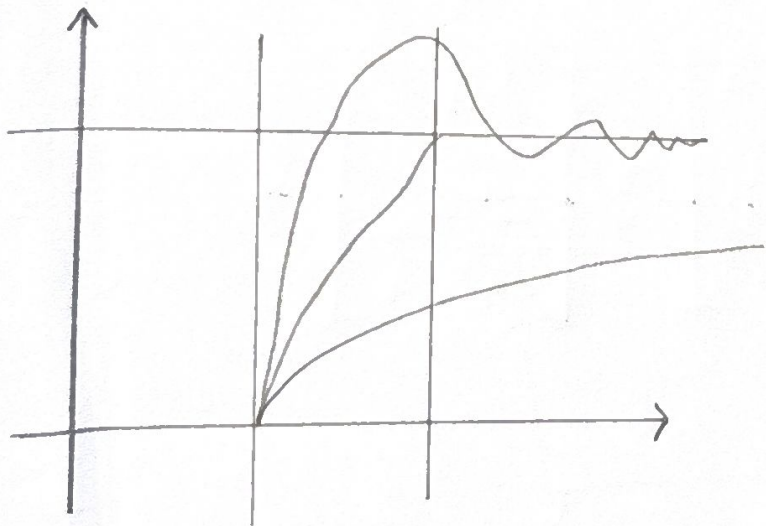
Root locus, Bode & Nyquist are Basic

Openloop

$$\Rightarrow G(s) = \frac{N_G(s)}{D_G(s)}$$

roots (D_G) in the left S plane

for
stability



Closed loop

$$G_{cl} = \frac{N_{cl}}{D_{cl}} = \frac{N_G N_H}{D_G D_H + N_G N_H} = \frac{N_G N_H}{D_G D_H + N_G N_H}$$

$$D_{cl} = 1 + G_H$$

$$= 1 + \frac{N_G N_H}{D_G D_H}$$

$$= \frac{D_G D_H + N_G N_H}{D_G D_H}$$

for stability $\rightarrow$ roots ($D_G D_H + N_G N_H$)
in left of S-plane

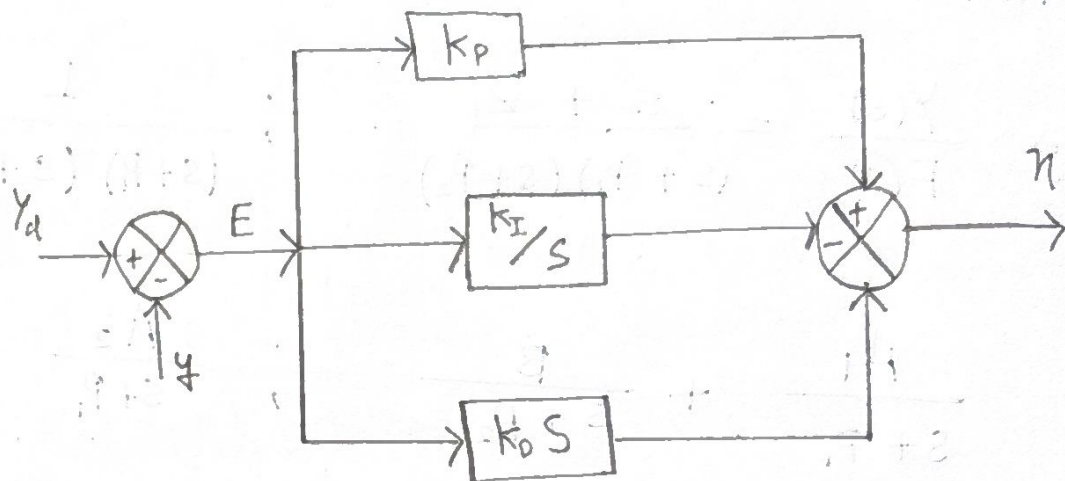
PID

$$\Rightarrow H = k_p$$

$$\Rightarrow H = k_p + \frac{k_I}{s}$$

$$\Rightarrow H = k_p T + k_D s$$

$$\Rightarrow H = k_p + \frac{k_I}{s} + k_D s$$



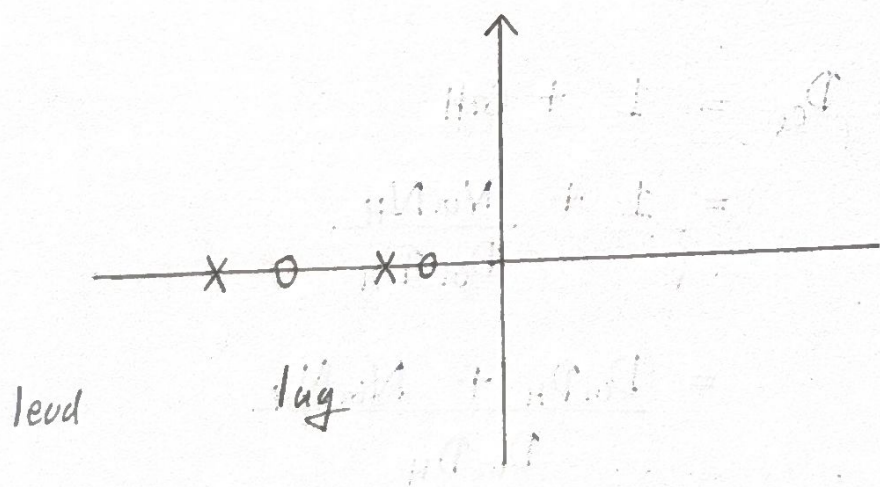
Lead-lag

$$\Rightarrow H = k_c \frac{s + \omega_z}{s + \omega_p}$$

$$H = \frac{k_D s^2 + k_D s + k_I}{s} = \frac{(s + z_1)(s + z_2)}{s}$$

$$\Rightarrow \text{If } |\omega_z| < |\omega_p| \Rightarrow \text{lead (Pole value is larger than zero)}$$

$$\Rightarrow |\omega_z| > |\omega_p| \Rightarrow \text{Lag (Zero " " " " pole)}$$



Date : 20/04/2026

$$\Rightarrow \dot{\bar{x}} = A\bar{x} + B\bar{n} \rightarrow \textcircled{A}$$

$$\Rightarrow \bar{y} = C\bar{x}$$

$\mathcal{L}[A]$

$$\Rightarrow s\bar{x}(s) - \bar{x}(0) = -A\bar{x}(s) + B\bar{n}(s)$$

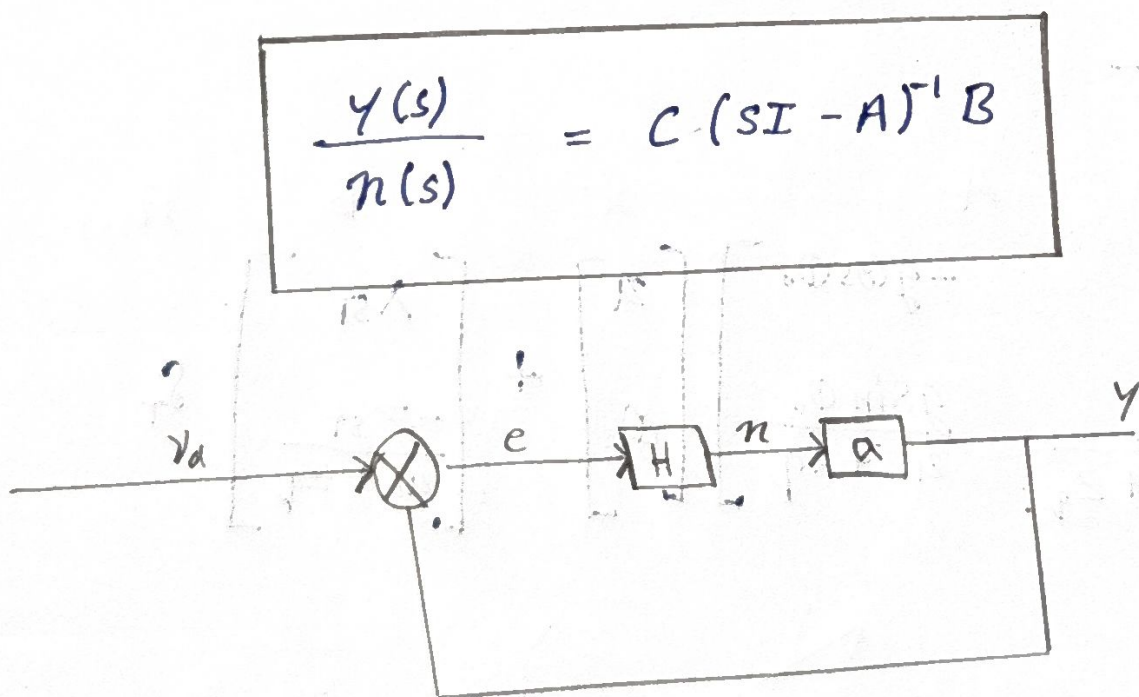
$$\Rightarrow \bar{y}(s) = C\bar{x}(s)$$

$$\Rightarrow (sI - A)\bar{x}(s) = \bar{x}(0) + B\bar{n}(s)$$

$$\Rightarrow (sI - A)\bar{x}(s) = B\bar{n}(s) \Rightarrow \bar{x}(s) = (sI - A)^{-1} B\bar{n}(s)$$

$$\Rightarrow \bar{y}(s) = C\bar{x}(s) = C(sI - A)^{-1} B\bar{n}(s)$$

$$\Rightarrow \frac{y(s)}{n(s)} = C(sI - A)^{-1} B$$



$$\Rightarrow H(s) = K_p + \frac{K_I}{s} + K_D s$$

$$= K_p \left[1 + \frac{T_i}{s} + T_D s \right]$$

⇒

$$T_i = \frac{k_I}{k_p}, \quad T_D = \frac{k_D}{k_p}$$

∴

$$H(s) = \frac{k_D s^2 + k_p s + k_I}{s}$$

⇒

$$h(t) = k_p e(t) + k_I \int e(t) dt + k_D \frac{de(t)}{dt}$$

⇒

$$\frac{Y_d}{Y} = \frac{GH}{1+GH}$$

$$1 + GH = 0 \rightarrow \textcircled{a} \quad (s) \dot{x} (A - I)$$

Roots of $\textcircled{a}$ should be stable

⇒

In root locus, $1 + G_k K_p = 0$, change the value of K_p and plot in the s plane.

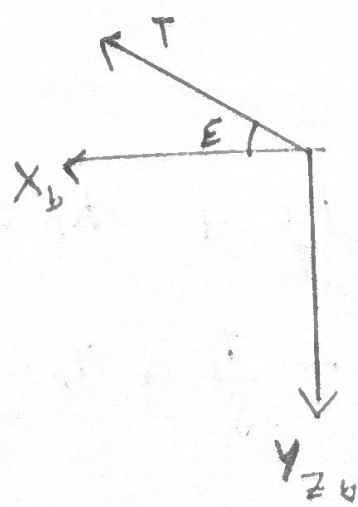
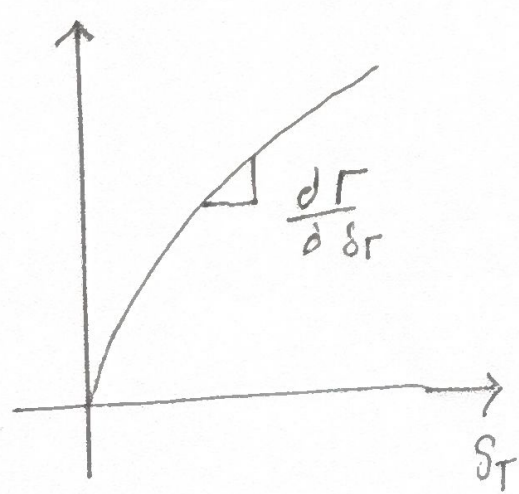
⇒

$$H(s) = K_p + \frac{k_I}{s}$$

⇒

$$\dot{\bar{x}} = \begin{bmatrix} \dot{z}_1 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} x_u \\ \frac{z_u}{u_0 + z_q} \end{bmatrix}$$

$$\begin{bmatrix} -g \cos \theta_0 \\ \frac{g \sin \theta_0}{u_0 + z_q} \end{bmatrix} \begin{bmatrix} z_1 \\ \theta \end{bmatrix} + \begin{bmatrix} x_{ST} \\ \frac{-z_{ST}}{u_0 + z_q} \end{bmatrix} = \dot{\bar{x}}_{ST}$$



$$T_{xb} = T \cos \epsilon$$

$$T_{zb} = T \sin \epsilon$$

⇒

$$\begin{bmatrix} u \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 0 \end{bmatrix}$$

⇒

$$\begin{bmatrix} \frac{u(s)}{sA(s)} \\ \frac{0(s)}{s_T(s)} \end{bmatrix} = \begin{bmatrix} s - X_u & g \cos \theta_0 \\ \frac{Z_u}{u_0 + Z_q} & \frac{s - g \sin \theta_0}{u_0 Z_q} \end{bmatrix}^{-1} \begin{bmatrix} X_{ST} \\ \frac{Z_{ST}}{u_0 + Z_q} \end{bmatrix}$$



We have to solve this now

Date :- 21/04/2026

$$\Rightarrow \begin{bmatrix} s - X_u & -g \\ \frac{Z_u}{u_0 + Z_q} & \frac{s}{u_0 + Z_q} \end{bmatrix} \begin{bmatrix} \frac{u(s)}{s_T(s)} \\ \frac{0(s)}{s_T(s)} \end{bmatrix} = \begin{bmatrix} X_{ST} \\ \frac{Z_{ST}}{u_0 + Z_q} \end{bmatrix}$$

$$\Rightarrow \Delta = \frac{(s - X_u)s + gZ_u}{u_0 + Z_q}$$

$$\Rightarrow = \frac{s^2 - X_us + gZ_u}{u_0 + Z_q}$$

$$\Rightarrow \frac{u(s)}{s_T(s)} = \frac{\Delta u}{\Delta}$$

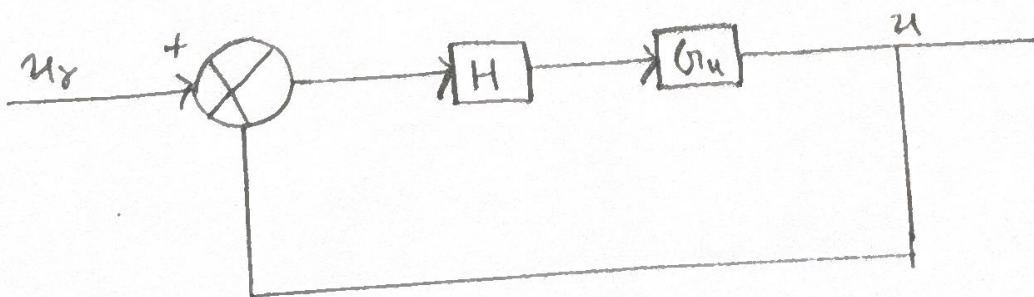
Cramer's Rule

$$\Rightarrow \Delta u = \frac{sX_{ST} + gZ_{ST}}{u_0 + Z_q}$$

$$\Rightarrow \frac{u(s)}{s_T(s)} = \frac{sX_{ST} + gZ_{ST}}{s^2 - X_us + gZ_u}$$

$$\Rightarrow \frac{0(s)}{s_T(s)} = \frac{(s - X_u)Z_{ST} - Z_uX_{ST}}{s^2 - X_us + gZ_u}$$

$$\Rightarrow G_u = \frac{u(s)}{s_T(s)} = \frac{sX_{ST} + Z_{ST}g}{s^2 - X_us + gZ_u}$$



$$\Rightarrow H(s) = K_p + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_I + K_p s}{s}$$

$$\Rightarrow \frac{H G_{ru}}{1 + H G_{ru}} = \frac{\left(\frac{K_D s^2 + K_I + K_p s}{s} \right) \left(\frac{s X_{sr} + g Z_{sr}}{s^2 - X_u s + g Z_u} \right)}{1 + \frac{(K_D s^2 + K_I + K_p s)(s X_{sr} + g Z_{sr})}{s(s^2 - X_u s + g Z_u)}}$$

$\Rightarrow 1 + G_{ru} H = 0 \Rightarrow$ all poles are in L s plane

$$\Rightarrow s(s^2 - X_u s + g Z_u) + (K_D s^2 + K_p s + K_I)(s X_{sr} + Z_{sr} g) = 0$$

$$\Rightarrow s^3 [K_D X_{sr} + 1] + s^2 [-X_u + K_D Z_{sr} g + K_p X_{sr}] + s [g Z_u + K_p Z_{sr} g + K_I X_{sr}] + K_I Z_{sr} g = 0$$

if we replace as $Z_{sr} g = Z_1$ & $X_{sr} = X_1$

$$\Rightarrow s(s^2 - X_u s + g Z_u) + (K_D s^2 + K_p s + K_I)(s X_1 + Z_1) = 0$$

$$\Rightarrow s^3 [K_D X_1 + 1] + s^2 [-X_u + K_D Z_1 + K_p X_1] + s [g X_u + K_p Z_1 + K_I X_1] + K_I Z_1 = 0$$

$$\left. \begin{array}{l} T_p = 50s \\ M_p = 70\% \end{array} \right\} \xi, \omega_n \text{ phugoid}$$

K_p, K_I & K_D Case
unknown here

$$\left. \begin{array}{l} T_{CL} \leq 10s \\ M_p < 20\% \end{array} \right\} \text{Closed loop Control}$$

$$\Rightarrow s^2 + 2 \xi_{LP} \omega_{nLP} s + \omega_{nLP}^2 = 0$$

$s = \text{atib}$

$$\Rightarrow S^2 + 2 \sum \omega_{nd} S + \omega_{nd}^2 = 0$$

$$\Rightarrow (S+1)(S+2) = 0 \quad (\text{we can not have this structure})$$

$$\Rightarrow S = -a_d \pm ib_d$$

$$\Rightarrow S = -p_i$$

$$\Rightarrow S^3 + C_1 S^2 + C_2 S + C_3 = 0$$

$$\rightarrow k_D Z_1 + K_P X_1 - X_2 = C_1$$

$$\Rightarrow q Z_2 + K_P Z_1 + K_I X_1 = C_2$$

$$\Rightarrow K_I + \cancel{K_P} Z_1 = C_3$$

$$(S+P_i)(S^2 + 2 \sum \omega_{nd} S + \omega_{nd}^2) = 0$$

$$S^3 + S^2(P_i + 2 \sum \omega_{nd}) + S(2 \sum \omega_{nd} P_i + \omega_{nd}^2) + P_i + \omega_{nd}^2 = 0$$

$$PI \rightarrow H = \frac{K_I + K_D S}{S}$$

$$P \rightarrow H = K_P$$

if we have $\neq$ lead lag system

$$\Rightarrow H = \frac{\omega_z}{\omega_p} \frac{S + \omega_z}{S + \omega_p}$$

Date: 22/04/2026

$$\Rightarrow \dot{u} = X_u u - g \cos \theta_0 \theta + \frac{1}{m} \frac{\partial X}{\partial \delta_T} \delta_T$$

$$\Rightarrow \dot{\theta} = \frac{-Z_u}{u_0 + Z_u} u - \frac{g \sin \theta_0 \theta}{u_0 + Z_g} - \frac{Z_{\delta_T}}{u_0 + Z_g} \delta_T$$

$$\Rightarrow \begin{bmatrix} s - X_u & g \\ \frac{Z_u}{u_0} & s \end{bmatrix} \begin{bmatrix} \frac{u(s)}{\delta_T(s)} \\ \frac{\theta(s)}{\delta_T(s)} \end{bmatrix} = \begin{bmatrix} X_{\delta_T} \\ -\frac{Z_{\delta_T}}{u_0} \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} X_u & -g \\ -\frac{Z_u}{u_0} & 0 \end{bmatrix}; B = \begin{bmatrix} X_{\delta_T} \\ -\frac{Z_{\delta_T}}{u_0} \end{bmatrix}$$

$$\Rightarrow \frac{u(s)}{\delta_T(s)} = \frac{1}{s^2 - X_u s - \frac{g Z_u}{u_0}}$$
$$\Rightarrow = \frac{0.9925s - 0.016}{s^2 + 0.0435s + 0.077}$$

$$\Rightarrow 1 + K G_u = 0$$

$$\Rightarrow 1 + K \frac{N_u}{D_u} = 0$$

$$\Rightarrow D_u + K N_u = 0$$

$$\Rightarrow \boxed{s^2 + 0.0435s + 0.077 + K^* 0.9325s - K^* 0.016 = 0}$$

$$\Rightarrow \xi = 0.077$$

$$\xi_d = 0.7 ; \omega_d = 2.77$$

$$s^2 + 2\xi_d \omega_d s + \omega_d^2 = 0$$

$$\Rightarrow \omega_n = 0.277$$

$$s^2 + 3.48785 + 7.61 = 0$$

from eqⁿ ①,

$$(932k + 0.043) = 3.87 \Rightarrow k = 4.106$$

$\Rightarrow$

$$\Rightarrow 0.077 - 0.016k = 7.67 \Rightarrow k = 474$$

So, eqⁿ ① will become,

$$\Rightarrow s^2 + 3.878 + 0.013 = 0$$

$$\Rightarrow \omega_n = 0.11$$

$$\Rightarrow \xi = 16.97$$

$$\Rightarrow H_1 = K_c \frac{s + \omega_z}{s + \omega_p}$$

$$\Rightarrow 1 + H_1 G_u = 0$$

$$\Rightarrow K_c = \frac{\omega_z}{\omega_p}$$

$$\Rightarrow 1 + K_c \frac{s + \omega_z}{s + \omega_p} \cdot \frac{N_p}{D_u} = 0$$

$$\Rightarrow s^3 + f_1(\omega_p)s^2 + f_2(\omega_p) + K_c = 0$$

Ziegler-Nichols frequency response of proportion control.

Date : 27/04/2026

Stability Augmentation System

Pure Pitching Motion,

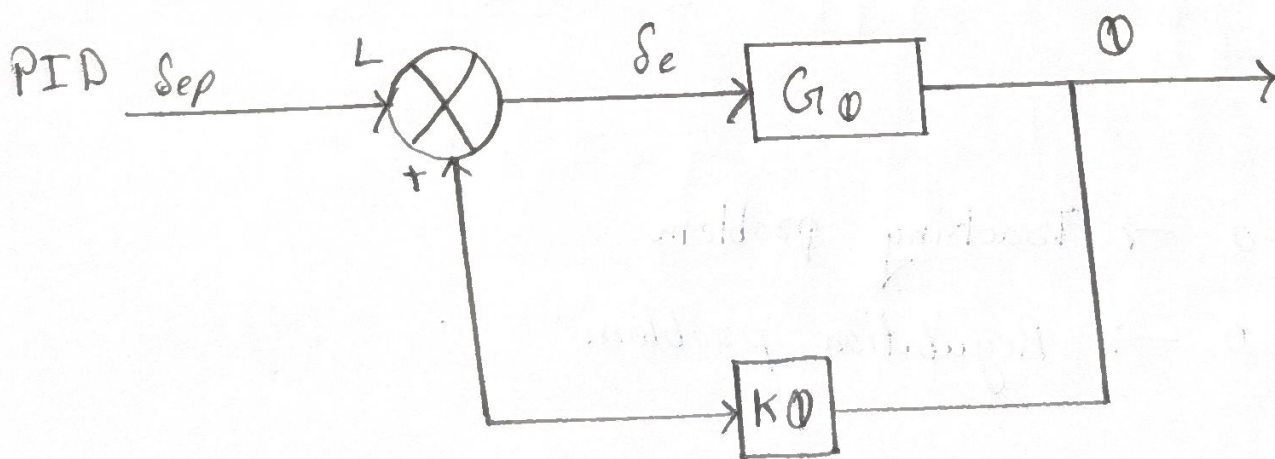
$$\Rightarrow I_y \ddot{\theta} = M$$

$$\Rightarrow \ddot{\theta} = \frac{M}{I_y} = M_\alpha \theta + M_q q + M_{\ddot{\alpha}} \ddot{\alpha} + M_{\delta e} \delta e$$

$$\Rightarrow \ddot{\theta} - (M_\alpha + M_{\ddot{\alpha}}) \theta - M_q q = M_{\delta e} \delta e = M_{\delta e} \delta_{ep} + \bar{M}_{\delta e} k_\theta \theta$$

$$\Rightarrow \ddot{\theta} - (M_q + M_{\ddot{\alpha}}) \dot{\theta} - (M_\alpha + \bar{M}_{\delta e} k_\theta) \theta = M_{\delta e} \delta_{ep}$$

$$\Rightarrow \delta e = \delta_{ep} + k_\theta \theta \quad / \quad \delta_{ep} = k_\theta \dot{\theta} + k_u \theta$$



Revise

State Space

$$\dot{\bar{x}} = A\bar{x} + B\bar{u}$$

$$\bar{y} = C\bar{x}$$

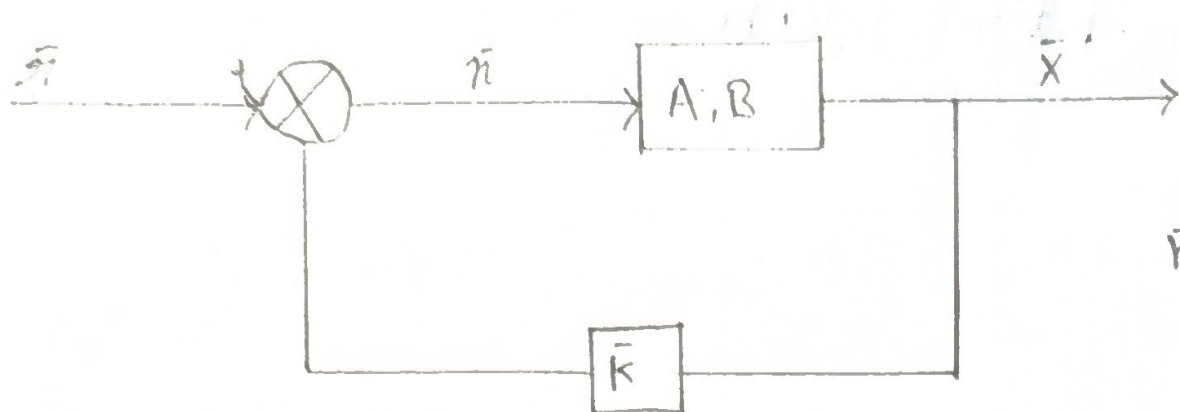
$$\bar{x} = \bar{C}_0 e^{-\lambda T}$$

$$\bar{x}(t) = e^{At} [\bar{x}(0) + \int_0^t e^{-As} B \bar{u}(s) ds]$$

$$e^{At} = \Phi(t) \rightarrow \text{State transition Matrix}$$

$$e^{At} = V e^{-\lambda t} V^{-1} \rightarrow [(sI - A)^{-1}]$$

(a) Full State feedback



$$\bar{n} = \bar{r} - \bar{K} \bar{x}$$

$$M = 0, E$$

(b) Observer feed back

$$\Rightarrow \bar{x} = [\alpha, q] \quad , \quad \alpha_d \rightarrow \bar{r}$$

$$\Rightarrow \bar{n} = [\delta e] \quad , \quad \bar{r}_\alpha = \alpha_d ; \bar{r}_q = q_d$$

$$\Rightarrow \bar{r} = [\alpha_d, q_d] \quad , \quad \delta e = \alpha_d - \bar{K} \begin{bmatrix} \alpha \\ q \end{bmatrix}$$

$$\Rightarrow \delta e = q_d - K \begin{bmatrix} \alpha \\ q \end{bmatrix}$$

$$0.1 + 0.21 = 0.21$$

{ Model predictive Control
LQR Can be Studied }

⇒ Ref. Signal $\bar{r} \neq 0 \rightarrow$ Tracking problem

⇒ $\bar{r} = 0 \rightarrow$ Regulation problem

* Regulator Problem :-

$$\Rightarrow \bar{n} = -\bar{K} \bar{x}$$

$$\Rightarrow \dot{\bar{x}} = A \bar{x} - B \bar{K} \bar{x}$$

$$\Rightarrow \dot{\bar{x}} = (A - B \bar{K}) \bar{x}$$

$$\Rightarrow \dot{\bar{x}} = \underline{A_a} \bar{x} \rightarrow \text{closed loop}$$

$$\Rightarrow \bar{x} = n \times 1, \bar{A} = \bar{n} \times \bar{n}, \bar{n} = n \times 1, \bar{B} = n \times m, \bar{k} = m \times n$$

* Tracking Problem :-

$$\Rightarrow \dot{\bar{x}} = (A - Bk)\bar{x} + B\bar{r}$$

* Formula for pole placement method :-

$$\Rightarrow A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}; B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}; k = [k_1, k_2]; h = 1 \times 1$$

$$\Rightarrow \lambda^n + C_1 \lambda^{n-1} + \dots$$

$$\Rightarrow \lambda^2 + C_2 \lambda + C_2 = 0$$

$$\Rightarrow \lambda^2 + 2 \xi \omega_n \lambda + \omega_n^2 = 0$$

* Ackerson's formula :-

$$\Rightarrow k = [0 \quad 0 \quad 0 \quad \dots \quad 1] P^{-1} \Delta_d(A)$$

$$\Rightarrow \Delta_d(A) = A^2 + 2 \xi \omega_n A + \omega_n^2 I$$

$$\Rightarrow \lambda^2 + 2 \xi \omega_n \lambda + \omega_n^2 = 0$$

$P \rightarrow$ Controllability matrix,

↓

if the rank is full then it's Controllable

$$\Rightarrow P = [B \quad AB \quad A^2 B \quad \dots \quad A^{n-1} B]$$

Date :- 28/04/2026

$$\Rightarrow A = \begin{bmatrix} -2.39 & 1 \\ -14.23 & -3.3 \end{bmatrix}, B = \begin{bmatrix} -7.59 \\ -28.92 \end{bmatrix}$$

$$\Rightarrow \zeta = 0.8 ; \omega_n = 4$$

$$\Rightarrow \dot{\bar{x}} = A\bar{x} + B\bar{r}$$

FSFB



$$\bar{r} = \bar{r} - k\bar{x}$$

$$\Rightarrow \dot{\bar{x}} = (A - Bk)\bar{x} + B\bar{r}$$

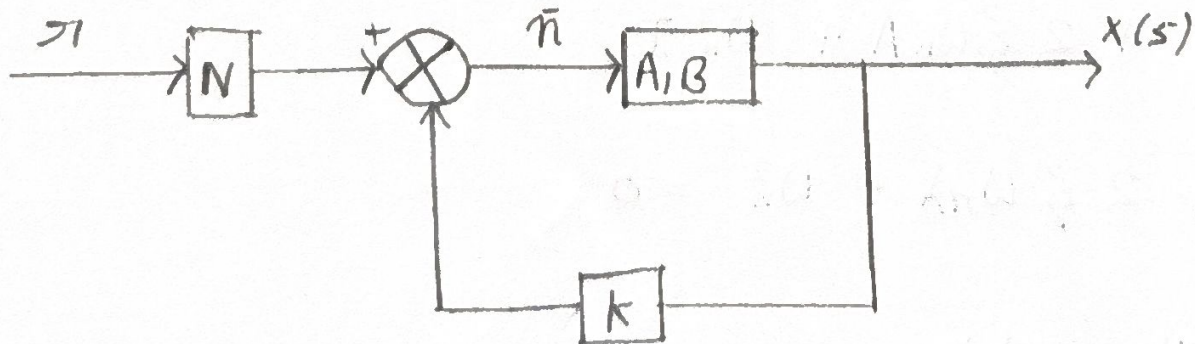
$$\Rightarrow S\bar{x}(s) = (A - Bk)\bar{x}(s) + B\bar{r}(s)$$

$$\Rightarrow \frac{\bar{x}(s)}{\bar{r}(s)} = [SI - A + Bk]^{-1} B$$

$$\Rightarrow t \rightarrow \infty \Rightarrow \bar{x} = \bar{r}$$

$$\Rightarrow \frac{\bar{x}(s)}{\bar{r}(s)} = \underbrace{[SI - A + Bk]^{-1} B}_N [(SI - A + Bk)^{-1} B]$$

$$= N(SI - A + Bk)^{-1} B$$



$$\Rightarrow A_a = \begin{bmatrix} a_1 - b_1 k_1 & a_2 - b_2 k_2 \\ a_3 - b_1 k_1 & a_4 - b_2 k_2 \end{bmatrix}$$

$$\Rightarrow |\lambda I - A| = \begin{vmatrix} \lambda - a_1 + b_1 k_1 & a_2 - b_2 k_2 \\ a_3 - b_1 k_1 & \lambda - a_4 + b_2 k_2 \end{vmatrix}$$

$$\Rightarrow = \lambda^2 + \lambda(k_1 k_2 + k_1 + k_2) + (k_1 k_2 + k_1 + k_2) = 0$$

$$\Rightarrow K = [0 \ 1] \cdot P^{-1} \Delta_d(A)$$

$$\Rightarrow \Delta_d(A) = A^2 + 6.4A + 16I$$

$$\Rightarrow P = \begin{bmatrix} -7.59 & [A] \begin{bmatrix} -7.59 \\ -28.92 \end{bmatrix} \end{bmatrix}$$

$$\Rightarrow \therefore P = \begin{bmatrix} -7.59 & -10.78 \\ -28.92 & 2.06 \end{bmatrix}$$

$$\Rightarrow P^T = \begin{bmatrix} -0.11 & -0.006 \\ -0.0154 & 0.009 \end{bmatrix}$$

$$\Rightarrow \Delta_d(A) = \begin{bmatrix} -7.81 & 0.61 \\ -8.71 & -8.43 \end{bmatrix}$$

$$K = [0.0852 \quad -0.0436]$$

Value
find $\uparrow$ theorem,

This is iterative
Solution

$$\Rightarrow S \rightarrow 0 \quad \frac{\bar{X}(s)}{\bar{Y}(s)} = S \frac{X(s)}{\bar{Y}(s)}$$

Rate Saturation :-

$$\Rightarrow \left| \frac{dn}{dt} \right| \text{ is finite.}$$

Optimal Control :-

Control a System such that there is a general Condition.

$$\Rightarrow J = \int_0^T [x^T Q x + n^T R n] dx$$

|
cost function

|
always positive definite

$$\dot{\bar{x}} = A \bar{x} + B \bar{n}$$

$$\dot{\bar{x}} = f(\bar{x}, \bar{n}, t)$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x_1^2 q_{11} + x_2 x_1 q_{12} + x_1 x_2 q_{21} + x_2^2 q_{22}]$$