Indian Institute of Space Science and Technology
Department of Avionics

AVD624/ESG667 Computer Vision Pre-Mid Exam

Total Marks: 20 **Duration:** 40 Minutes
Questions: 4 **Marks per Question:** 5

Instructions: Answer **all** questions. No extra sheets will be provided. Marks for each sub-part are indicated on the right. All the best!

Question 1. Homography Estimation via Direct Linear Transform (DLT)

Consider the following four point correspondences between two planes:

$$x_1 = (0,0)^\top \leftrightarrow x_1' = (1,1)^\top, \quad x_2 = (1,0)^\top \leftrightarrow x_2' = (4,1)^\top,$$
$$x_3 = (1,1)^\top \leftrightarrow x_3' = (4,4)^\top, \quad x_4 = (0,1)^\top \leftrightarrow x_4' = (1,4)^\top.$$

(a) For a single correspondence $(x, y) \leftrightarrow (x', y')$, the DLT formulation gives a 2×9 matrix A_i such that $A_i\,\mathbf{h} = \mathbf{0}$, where $\mathbf{h} = \mathrm{vec}(H)$ is the vectorisation of the homography. Write the explicit A_i for the first correspondence $x_1 \leftrightarrow x_1'$. **[1 marks]**

(b) State the minimum number of correspondences required to solve for H via DLT. Specify the geometric constraint that the chosen points must satisfy. **[1 mark]**

(c) After stacking all A_i into matrix A and solving $A\mathbf{h} = \mathbf{0}$ via SVD, which singular vector yields $\hat{\mathbf{h}}$? Justify why that choice minimises $\|A\mathbf{h}\|$ subject to $\|\mathbf{h}\| = 1$. **[3 marks]**

Question 2. Camera Projection and Zhang's Calibration Method

A calibrated camera has the intrinsic matrix

$$K = \begin{pmatrix} 500 & 0 & 320 \\ 0 & 500 & 240 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) A 3-D world point $\mathbf{X}_w = (1,\,0,\,5)^\top$ (metres) is projected with $R = I$ and $\mathbf{t} = \mathbf{0}$. Write the full projection equation $\lambda\,\tilde{\mathbf{x}} = K\,[R \mid \mathbf{t}]\,\tilde{\mathbf{X}}_w$ and compute the pixel coordinates (u, v). **[2 marks]**

(b) The camera is now translated to $\mathbf{t} = (-1,\,0,\,0)^\top$ with $R = I$ unchanged. Recompute (u, v). Using both results, explain how lateral translation affects pixel displacement and how the focal length f mediates this relationship. **[2 marks]**

(c) In Zhang's method the calibration target is a planar checkerboard with $Z = 0$. Write the simplified projection matrix under this assumption, and state in one sentence why this planar target is sufficient to recover all five intrinsic parameters. **[1 mark]**

Question 3. Linear Shift-Invariant Systems and Image Filtering

Consider the 5×5 image patch $f[n, m]$ and 3×3 box filter $h[n, m]$ below (top-left pixel is index $[0, 0]$):

$$f = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 90 & 90 & 90 & 0 \\ 0 & 90 & 0 & 90 & 0 \\ 0 & 90 & 90 & 90 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad h = \frac{1}{9} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

(a) Compute the 2-D convolution output $g[2, 2]$ (the centre pixel). **[1.5 marks]**

(b) Determine whether the moving-average filter h is an **LTI** system by explicitly verifying both required properties:
(i) superposition and (ii) time-invariance. **[2 marks]**

(c) Give **two specific advantages** of a Gaussian filter over the uniform box filter h for image smoothing. **[1.5 marks]**

Question 4. Edge Detection: Image Gradients and the Canny Detector

(a) The Canny pipeline includes a step called **Non-Maximum Suppression (NMS)**.

(i) Describe precisely what NMS does and why it is necessary after computing gradient magnitudes. **[1 mark]**

(ii) A pixel at (x, y) has gradient magnitude $|\nabla G| = 80$ and gradient direction $\theta = 90°$. Its two neighbours along the gradient direction have magnitudes 70 and 85. Does this pixel survive NMS? **[0.5 marks]**

(b) Canny uses **hysteresis thresholding** with $T_H = 80$ and $T_L = 40$. Classify each pixel and state the final edge decision for Pixels B and C:

Pixel	**Gradient Magnitude**	**Connectivity**
A	120	—
B	55	Connected to Pixel A
C	55	Not connected to any strong edge pixel

[1.5 marks]

(c) The Sobel operator approximates the image gradient using two 3×3 kernels.

(i) Write the standard Sobel kernels G_x and G_y. **[1 mark]**

(ii) Given gradient components $G_x = 3$ and $G_y = 4$ at a pixel, compute the gradient magnitude $|\nabla I|$ and the gradient direction θ (in degrees). **[1 mark]**

— *End of Question Paper* —