Indian Institute of Space Science and Technology Trivandrum

I SEMESTER , 2022

ExamType: Quiz 1

DEPARTMENT OF AVIONICS

computer vision/ computer vision and advanced image processing

(Time allowed: ONE hours)

NOTE: Read all questions first. **There are questions worth 30 marks.** If something is missing in a problem description, clearly mention your assumptions with your solution. If require, use sketches to illustrate your findings.

1. Explain what are the key ideas that are used in edge detection. What all things can cause an edge in an image. Name a few methods to detect edges in the image. How Sobel edge detection method works. (5 marks)

2. The image below is an image of a 3 pixel thick vertical line. (5 marks)

   (a) Show the resulting image obtained after convolution of the original with the following approximation of the derivative filter $[-1; 0; 1]$ in the horizontal direction. How many local maxima of the filter response do you obtain ?

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

   (b) Suggest a filter which when convolved with the same image would yield a single maximum in the middle of the line. Demonstrate the result of the convolution on the original image.

   (c) What is a difference between Gaussian smoothing and median filtering? How would you decide to use one vs. another ?

3. Given two lines in the image denoted by their projective coordinates l_1 and l_2, how would you compute an intersection point of these two lines ? (1 mark)

4. Show that the line through two 2D points x and x' is $l = x \times x'$. (1 mark)

5. Under homography, how can we write the transformation of points in 3D from camera 1 to camera 2? Explain (mention all the steps needed) how do we estimate the homography. (3 marks)

6. What does it mean for the 2D convolution kernel (filter) to be separable ? (1 mark)

7. Which of the following statements are true of a pinhole camera? Which are false? (1 mark)

   (a) (A) Images in a pinhole camera are upside down.

   (b) (B) A pinhole camera has a fixed focal length,f.

   (c) (C) Images in a pinhole camera are a perspective projection

8. (write all that apply) Which of the following could affect the intrinsic parameters of a camera? (1 mark)

   (a) A crooked lens system.

   (b) Diamond/Rhombus shaped pixels with non right angles.

   (c) The aperture configuration and construction.

   (d) Any offset of the image sensor from the lenss optical center.

9. state true or false (with a proper reasoning)

   (a) Pincushion distortion modifies the image only along the vertical direction and barrel distortion modifies the image only along the horizontal direction. (1 mark)

   (b) The vanishing point associated with a line in 3D space (when viewed through a pinhole camera) can never be a point at infinity (1 mark)

   (c) The result of applying a scale, rotation, and translation (in that order) to a vector **v** is equal to the result of applying the same rotation, translation, and scale (in that order) to the same vector **v**. (1 mark)

   (d) It is impossible to estimate the intrinsic parameters of the camera given a single image even with prior information about the scene (1 mark)

10. Which of the following always hold(s) under affine transformations? (1 mark)

    (a) Parallel lines remain parallel

    (b) Ratio of lengths of parallel line segments remain the same

    (c) Ratio of areas remain the same

    (d) Perpendicular lines remain perpendicular

    (e) Angles between two line segments remain the same

11. The figure (1) below shows the outputs of applying one of transformations (projective, affine, similarity, and isometric) to a square with vertices at (1,1), (1,-1), (-1,-1), (-1,1) tell which is the most specific transformation used to generate each output. if possible write the transformation matrix and write the degrees of freedom. (2 marks)

12. Suppose we want to solve for the camera matrix K and that we know that the world coordinate system is the same as the camera coordinate system. Assume the matrix K has the structure outlined below. Note that K_{33} is an unknown. Assume that we are given n correspondences.

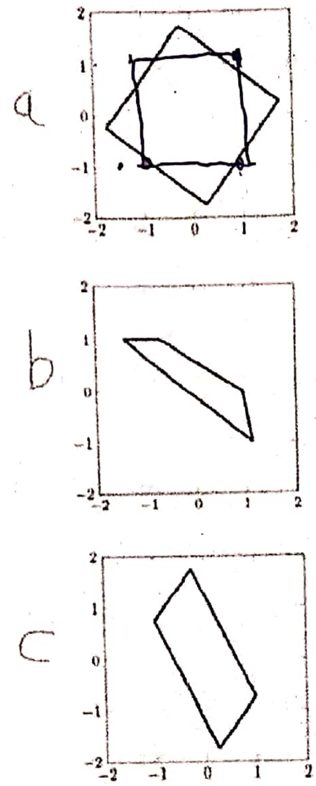

Figure 1: figure for understanding the various transfrom (a) (b) and (c) top-to-bottom respectively)

Each correspondence consists of a world point (x_i, y_i, z_i) and its projection (u_i, v_i) for $i = 1, ..., n$. (5 marks)

$$\begin{pmatrix} K_{11} & K_{12} & K_{13} \\ 0 & K_{22} & K_{23} \\ 0 & 0 & K_{33} \end{pmatrix}$$

(a) What is the minimum number of correspondences needed to solve for the unknowns in the matrix K?

(b) Set up an equation of the form $Ax = 0$ to solve for the unknowns in K (where A is a matrix, and x and 0 are vectors). Be specific about what the matrix A and vector x are.

(c) Explain how to solve for the unknowns in the camera matrix K. Make sure $K33 = 1$.

Indian Institute of Space Science and Technology
AVD871 - Applied Markov Decision Processes and Reinforcement Learning
Department of Avionics

Quiz 1 for **B.Tech Semester VI, M.Tech Semester II** on **23/02/2022**

Note to the student

1. There are **5 questions** in this question paper on **1 page**, for a total of **20 marks.**
2. Answer **all** questions.

Question 1 (2 marks): Suppose a discrete time Markov chain is time-homogeneous (i.e., the transition probabilities are independent of the discrete time) and has the following transition probability matrix

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0.2 & 0.1 & 0.4 & 0.3 \\ 0.3 & 0.1 & 0.1 & 0.5 \\ 0.7 & 0.1 & 0.1 & 0.1 \end{bmatrix}$$

Draw the transition diagram corresponding to this transition probability matrix.

Question 2 (3 marks): Suppose $X_0, X_1, X_2, \cdots$ is a discrete time random process. Suppose you observe one run of the above random process and observe the following sequence.

$$0, 1, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 2, 3, 3, 2, 3, 3, 3, 2, 2, 3, 2, 3, 3, 2, 2$$

Suggest a discrete time Markov model for the above random process.

Question 3 (3 marks): Suppose $X_0, X_1, X_2, \cdots$ is a discrete time Markov chain. Consider a new discrete time random process $Y_0, Y_1, Y_2, \cdots$ where each $Y_i = g(X_i)$ and $g : \mathbb{R} \to \mathbb{R}$. Verify whether the random process $Y_0, Y_1, Y_2, \cdots$ is a discrete time Markov chain or not?

Question 4 (2 marks): Suppose X and Y are two independent random variables. Prove that $\mathbb{E}XY = \mathbb{E}X\mathbb{E}Y$.

Question 5 (10 marks): Suppose $(X_t, t \in \{0, 1, 2, \dots\})$ is a discrete time Markov chain with state space $\mathcal{S} = \{0, 1, 2, 3 \dots, m\}$, a transition probability matrix $\mathcal{P}$, and initial distribution $P(X_0 = x_0)$.

1. Using pseudocode, write down a function that will simulate one run (or one sample function) of this Markov chain for a duration of 100 time steps.
2. Using pseudocode, and using the function that you have written in (1) above, show how you can obtain the conditional probability $P(X_{40} = i, X_{64} = j | X_0 = k, X_1 = m)$ from simulation. Here i, j, k, m are elements of $\mathcal{S}$ and are assumed to be user defined inputs.

Indian Institute of Space Science and Technology

Thiruvananthapuram - 695 547

Quiz I - February 2022

M.Tech (Machine Learning and Computing, Geoinformatics) - II Semester

MA625 - Statistical Models and Analysis

Date : 22/02/2022 **Time: 09.00 am to 10.00 am** **Maximum Marks: 20**

ALL questions carry EQUAL marks.

1. Is the function f defined by

$$f(x) = \begin{cases} \dfrac{x+1}{\theta(\theta+1)} e^{-x/\theta}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\theta > 0$ a pdf? If so, find the corresponding distribution function. Also, compute $P\{X \geq 1\}$.

2. For the bivariate random variable (X, Y) with pdf

$$f(x, y) = \frac{1}{4}\left[1 + xy(x^2 - y^2)\right], |x| \leq 1, |y| \leq 1,$$

find the marginal pdf's of X and Y. Also, find the conditional pdf of Y given $X = x$.

3. If $\overline{X}$ is the mean of a random sample of size n taken from a population having mean μ and variance σ^2, find $E(\overline{X})$ and $\text{var}(\overline{X})$. Also, prove that $\overline{X}$ converges to μ in probability.

4. Define chi-square distribution. If S^2 is the variance of a random sample of size n taken from a normal population having finite variance σ^2, find the probability distribution of $\dfrac{(n-1)S^2}{\sigma^2}$. Also, find $\text{var}(S^2)$ and show that it decreases as n increases.

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Indian Institute of Space Science and Technology – Thiruvananthapuram

M.Tech. Quiz-I

MA624 Advanced Machine Learning

Date: 21 February 2022 | Time: 1 hour | Max. Marks: 30

Answer all questions

1. (a) Consider a convergent sequence (x_n), which converges to x_0. For $\epsilon = 0.5$, $d(x_m, x_0) < \epsilon, \forall m > 100$. Find an appropriate k such that $d(x_m, x_{m'}) < k, \forall m, m' > 100$. (2)

   (b) Consider a continuous function: $T : (X, d) \to (Y, \tilde{d})$. T is continuous at x_0. $\delta = 0.2$ corresponding to $\epsilon = 0.5$. Where does the image of points in $B(x_0, 0.1)$ lie as far as $B(Tx_0, 0.5)$ is concerned? Justify your answer. (2)

2. Consider the EEG signals $\{x_i\}, i = 1, 2, \ldots 50$. Each of these x_i is a combination of 8 signals of which 5 are genuine and 3 are artifacts. Using these facts, write the mathematical formulation of ICA and specify the size of mixing matrix. Describe a procedure to find the artifacts of the signals. (5)

   $x_0 \to M$ $\quad d(x_n) < \epsilon$

3. (a) Consider metric space $M = [-1, 3] \cup \{100, 233\}$. Find the limit points of M.
   $(-1, 0, 1, 2, 3, 100, 233)$ (1.5)

   (b) M is a dense subset of a metric space X. Prove that for every $x \in X$, $\exists$ a sequence $(x_n) \in M$ such that x_n converges to x. (3)

   (c) Find $\|x^2 + x\|$, using the norm defined in $L^2[0, 1]$. (1.5)

4. (a) Consider a HMM model. Let $S = \{N_1, N_2\}$. Let $V = \{P_1, P_2\}$. Let $a_{1,1} = 0.6, a_{2,2} = 0.7, b_{N_1}(P_1) = 0.6, b_{N_2}(P_2) = 0.3$ and $\pi_{N_1} = 0.4$.

   i. Using Viterbi algorithm find the optimal state sequence that generates the observation sequence $P_2 P_1 P_2$. (5)

   ii. Find $\beta_2(N_2)$, where $O = (P_1, P_1, P_1)$. (2)

   (b) Explain an algorithm to estimate the parameters of a HMM model. (4)

5. On the basis of the given data, explain the procedure of finding 2 clusters using Gaussian mixture model clustering. Data is: $x_1 = (-1, 1.0)^T, x_2 = (2, 0, -1)^T, x_3 = (1, 1, 2)^T$. (4)

Indian Institute of Space Science and Technology – Thiruvananthapuram

M.Tech. Quiz-I

MA873 Graphical and Deep Learning Models

Date: 28 February 2022 | Time: 1 hour | Max. Marks: 30

Answer all questions

1. Consider a deep learning model with 4 hidden layers. The number of neurons in layer 1 is 3, layer 2 is 2, layer 3 is 10, layer 4 is 20. The data modeling task is 5 class classification problem. The data point is $x_i \in \mathbb{R}^{1000}$.

   (a) Write the size of the weight matrix and bias vector in each layer. (3)

   (b) What cost function and output activation function would you choose for the model? Justify your answer. (2)

   (c) Express the model as a composite function. (3)

2. Consider a deep learning model with following architecture: The data is $x = (0, 1, -3)^T$, $y = 1$. The current parameters in each layer are as follows.

$$W^{(1)} = \begin{bmatrix} .2 & 2 & 1 \\ .3 & -2 & 1 \end{bmatrix}, \quad b^{(1)} = \begin{bmatrix} .2 \\ .3 \end{bmatrix}, \quad W^{(2)} = \begin{bmatrix} .01 & .2 \\ .001 & .02 \\ 1 & .3 \end{bmatrix}, \quad b^{(2)} = \begin{bmatrix} .1 \\ .01 \\ .2 \end{bmatrix}$$

$W^{(3)} = [1, -1, 2], b^{(3)} = .3$

Hidden function: ReLU. The task is regression. Dropout in the hidden layers: $r^{(1)} = (0, 1)^T, r^{(2)} = (1, 0, 1)^T$. Find the loss $(\mathscr{L}(f(x_i), y_i))$. Using the given information, find the updated values of the parameters by applying SGD (learning parameter $\alpha = 0.01$). (5)

3. (a) Consider a CNN model: X is the Input and F is the filter

$$X = \begin{bmatrix} 1 & -1 & 0 & 2 & -3 \\ 2 & 3 & 1 & 0 & -1 \\ 1 & -2 & 1 & -2 & 1 \\ 1 & 0 & -1 & 1 & -2 \end{bmatrix}, F = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$$

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Stride: $S = 2$, Padding: $P = 1$, Activation function: ReLU, Pooling: Max pooling with stride S =1. Find the output. (4)