Column x row   3 x 3 x 3 x 1

Indian Institute of Space Science and Technology Trivandrum

I SEMESTER , 2026

ExamType: Quiz 1

DEPARTMENT OF AVIONICS

computer vision/ computer vision and advanced image processing

(Time allowed: ONE hours)

NOTE: Read all questions first. **There are questions worth 25 marks.** If something is missing in a problem description, clearly mention your assumptions with your solution. If required, use sketches to illustrate your findings.

1. A pinhole camera projects a 3D point P, located at coordinate $P = [1, 0, 0]^T$ in the world coordinate system, onto the image at point p using the projection matrix M, as

$$p = MP$$

where

$$M = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Intrinsic matrix } K} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{Camera matrix } [I|0]} \underbrace{\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{World-to-camera transform } T_{cw}}$$

3 x 3   4 x 3   4 x 4

(a) Compute the row and column of p in the image. (2 marks)

(b) Compute the following camera parameters from the intrinsic matrix K: (3 marks)

- (i) Focal length
- (ii) Skew
- (iii) Offset along x-axis
- (iv) Offset along y-axis
- (v) Scaling

2. Corner Detection

(a) We have discussed corner detection for 2D images. First, briefly describe the Harris corner detection method for 2D images. (1.5 marks)

(b) Given $M = \begin{bmatrix} 8 & 3 \\ 3 & 5 \end{bmatrix}$, compute eigenvalues and Harris response with $k = 0.06$. (Hint: use trace trick) for computing the corner response. (1.5 marks)

(c) Now, we want a method for corner detection for use with 3D images, i.e., there is an intensity value for each (x,y,z) voxel. Describe a generalization of either the Harris corner detector by giving the main steps of an algorithm, including a test to decide when a voxel is a corner point (2 marks)

3. Homography

(a) Define homography. Under what geometric conditions does a homography exist between two images? (0.5 marks)

(b) Show that a homography has 8 degrees of freedom. (0.5 marks)

(c) How many point correspondences are required to compute a homography? Justify your answer. (1 mark)

(d) Given four point correspondences between two images of a planar surface: (3 marks)

$$(0,0) \leftrightarrow (100,200)$$
$$(1,0) \leftrightarrow (200,200)$$
$$(1,1) \leftrightarrow (200,300)$$
$$(0,1) \leftrightarrow (100,300)$$

(i) Derive the Direct Linear Transform (DLT) formulation for estimating a homography.

(ii) Construct the linear system $Ah = 0$.

(iii) Compute the homography matrix H.

4. Image Processing in Spatial Domain

(a) Why is convolution commutative while correlation is not? Prove that using one example or through derivation. (1 mark)

(b) We know that template matching is performed using cross-correlation. Given a 4×4 image I:

$$I = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 50 & 60 & 70 & 80 \\ 90 & 100 & 110 & 120 \\ 130 & 140 & 150 & 160 \end{bmatrix}$$

and a 2×2 template T:

$$T = \begin{bmatrix} 60 & 70 \\ 100 & 110 \end{bmatrix}$$

Compute the matching score at the top-left position $(0,0)$ using:

(i) Sum of Squared Differences (SSD) (1.5 marks)

(ii) Normalized Cross-Correlation (NCC) (1.5 marks)

(c) We discuss in the class about integral image (also called summed area table) for a 2D grayscale image $I(x,y)$. Given the image: (1 mark)

$$I = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

(i) Compute the integral image S.

(ii) Using S, compute the sum of the rectangular region bounded by top-left $(1,1)$ and bottom-right $(2,2)$ (zero-based indexing, i.e. Pixel value 1 is at position $(0,0)$).

5. Image Processing in Frequency Domain. Consider the 1D Gaussian function:

$$g(x) = e^{-ax^2}, \qquad a > 0$$

The continuous-time Fourier Transform is defined as:

$$G(\omega) = \int_{-\infty}^{\infty} g(x)\, e^{-i\omega x}\, dx$$

(a) Show that the Fourier Transform of a Gaussian is also a Gaussian. (1.5 marks)

(b) Explain why a narrow Gaussian in the spatial domain produces a wide Gaussian in the frequency domain. Relate your answer to the uncertainty principle. (1.5 marks)

(c) Given the 2D Gaussian: (2 marks)

$$g(x,y) = e^{-a(x^2+y^2)}$$

Show that its 2D Fourier Transform is:

$$G(u,v) = \frac{\pi}{a} e^{-\frac{u^2+v^2}{4a}}$$

Hint: Use separability of the 2D Fourier Transform.

6. The Laplacian operator in continuous 2D is defined as:

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

In discrete form, a commonly used Laplacian mask is:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Answer the following questions:

(a) The Laplacian filter is often described as an *isotropic* operator. Explain what isotropy means in this context and justify mathematically why the Laplacian is rotation invariant. (1 mark)

(b) The Laplacian is considered a high-pass filter. (2 marks)
Explain this in both:

(i) Spatial domain intuition

(ii) Frequency domain interpretation

(c) In edge detection, why is the Laplacian often combined with Gaussian smoothing (LoG)? Explain what would happen if smoothing were not performed. (1 mark)

(d) The Laplacian produces zero response in uniform intensity regions. Prove this using the discrete mask and explain its significance in edge detection. (1 mark)