# Indian Institute of Space Science and Technology

## AE721/AE480 Boundary Layer Theory

**Midterm Exam**

(March 04, 2025)

**Duration: 120 min** **Total marks: 60**

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**General Instructions:**

- **Draw schematic diagram wherever possible and label clearly**
- **State the assumptions clearly wherever required**
- **Assume suitable value for parameter if required and mention it clearly with units**

**Questions:**

1. Give brief answer
   (a) What is Galilean invariance and objectivity? Give examples. [3]
   (b) What is bulk viscosity? and its consequences. [3]
   (c) Compare diffusive transport with advective transport. [3]
   (d) What are the sources for circulation generation in a fluid flow? [3]

2. Write the dimensionless version of Prandtl's incompressible boundary layer equations and its assumptions and boundary conditions. Discuss the important implications compared to Navier-Stokes equations. [12]

3. Write the following terms in symbolic notation and briefly explain the physical meaning. Where, $\sigma_{ij}, u_j, w_2$, and $S_{ij}$ are stress tensor, velocity, vorticity and symmetric part of velocity gradient tensor, respectively. [10]

   $(a)\ \frac{d\sigma_{ij}}{dx_j}$, $(b)\ \frac{d(\sigma_{ij}u_j)}{dx_j}$, $(c)\ 2\mu S_{ij}S_{ij}$, and $(d)\ w_2\frac{du_1}{dx_2}$ $\left[w_j \frac{du_i}{\delta x_j}\right]$

4. Using tensor indicial notation, prove the following: [7]
   (a) $(\underline{\mathbf{a}} \times \underline{\mathbf{b}}) \times \underline{\mathbf{c}} = (\underline{\mathbf{a}} \cdot \underline{\mathbf{c}})\mathbf{b} - (\underline{\mathbf{b}} \cdot \underline{\mathbf{c}})\underline{\mathbf{a}}$
   (b) $\underline{\underline{\mathbf{M}}}\,\underline{\mathbf{b}} = (\underline{\mathbf{m}} \times \underline{\mathbf{b}})$ where, $\underline{\underline{\mathbf{M}}}$ is skew-symmetric tensor corresponding to $\underline{\mathbf{m}}$

5. Derive the rate of change of entropy of a material element and explain each term. Also, explain how thermodynamic II law is imposed in the fluid flow. [7]

6. The non-dimensional velocity field with respective to the stationary frame is given by $\mathbf{U} = 3x_1^2x_2\,\mathbf{e_1} + 2x_2^2\,\mathbf{e_2}$ ($\mathbf{e_1}$ and $\mathbf{e_2}$ are unit vectors in $x_1$ and $x_2$ direction). Calculate the following at point $A = (1, 1)$ [12]
   (a) the components of the velocity gradient $\frac{du_i}{dx_j}$, the rate of linear strain, the rate of shear strain, and vorticity at $A$,
   (b) What will be the linear strain rate, shear strain rate and vorticity at point A with respect to a person who rotates with angular velocity (non-dimensional) of $10\,\mathbf{e_3}$.

*⋆ End of Paper ⋆*